Worksheet 5: Vectors in Kinematics

Objectives

- Represent position, velocity and acceleration as vectors.
- Work with ballistic trajectories in the vertical and horizontal directions.

Summary

Non-constant acceleration

$v = v_0 + \int_0^t a dt$; $x = x_0 + \int_0^t v dt$

Position, velocity, and acceleration vectors

Position $\vec{r} = (x, y, z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Velocity $\hat{v} = \frac{\partial \hat{r}}{\partial t} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$

Speed $v = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \left(\frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\right) = \left(\frac{d^2x}{dt}, \frac{d^2y}{dt}, \frac{d^2z}{dt}\right) = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$

Component of $\vec{a}$ parallel to $\vec{v}$: $a_{\parallel}$ = rate of change of speed = dv/dt

Component of $\vec{a}$ perpendicular to $\vec{v}$: $a_{\perp}$. Affects direction of $\vec{v}$ only.

Projectiles

When the only force is gravity (no air resistance, etc.), the horizontal (x) and vertical (y) components of the motion can be considered independently. For a projectile launched from (x_0, y_0) with initial speed v_0 at angle θ above horizontal, the initial velocity $\vec{v}_0$ = (v_{0x}, v_{0y}) = ($v_0 \cos\theta$, $v_0 \sin\theta$) and

$x = x_0 + v_{0x}t$	$v_x = v_{0x}$	$a_x = 0$
$y = y_0 + v_{0y}t - {}^1/_2\, gt^2$	$v_y = v_{0y} - gt$	$a_y = -g$

This requires that the $+y$ direction be up.

Range Equation

The horizontal distance traveled by a projectile before landing at its launch height is $v_0^2 \sin(2\theta)/g$.

Problems

A projectile is launched from the origin at speed v_0 and angle θ above the horizontal.

a. Find the formula for the maximum height reached by the projectile.

b. Find the formula for the time at which the projectile reaches the top of its arc.

c. Find the formula for the horizontal distance the projectile travels to the top of its arc.

b. The projectile is a sofa fired from a hobbyist's catapult at a speed of 20 m/s at an angle of 50° above horizontal. Where (horizontal and vertical displacement from launch) is the top of its arc?

c. What is the sofa's speed at the top of its arc?