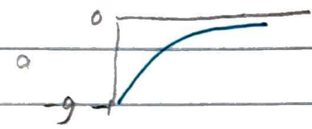
Position and Velocity by Integrating Acceleration

Exponential decay of acceleration

$a(t) = -ge^{-kt}$; $v_0 = 0$, $x_0 = H$ [k must be a reciprocal time]

Functional form



asymptotically approaches $a = 0$

$dv/dt = -ge^{-kt}$

$dv = -ge^{-kt}\,dt$

$v = \int dv = -g \int e^{-kt}\,dt$ Substitute $u = -kt$

$du/dt = -k$, so $dt = -\frac{1}{k}du$

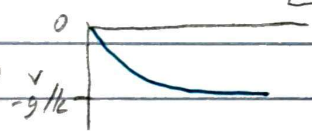
$v = -g \int e^{u}\left(-\frac{1}{k}du\right) = +\frac{g}{k}\int e^{u}du = g/k\, e^{u} + C = g/k\, e^{-kt} + C$

Incorporate initial condition $v_0 = 0$

$0 = g/k\, e^{-0} + C = g/k + C$

$C = -g/k$ so $\boxed{v = g/k\, e^{-kt} - g/k} = g/k\,(e^{-kt} - 1)$

Form



Position

$dx/dt = g/k\, e^{-kt} - g/k$

$x = \int dx = g/k \int e^{-kt}dt - g/k \int dt$

$x = g/k\left(-\frac{1}{k}\right)e^{-kt} - gt/k + C$ at $t = 0$, $x = H$, so

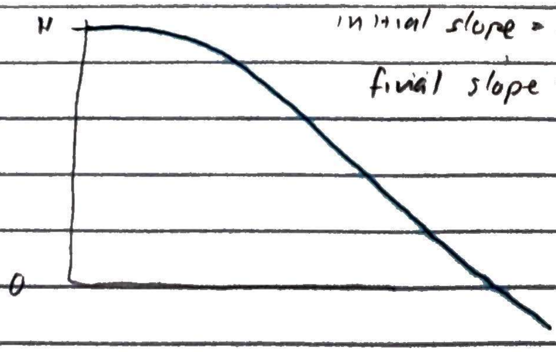
$H = -g/k^2 + C$

$C = H + g/k^2$ units work

$x = -g/k^2\, e^{-kt} - g/k\, t + H + g/k^2$

$\boxed{x = g/k^2\,(1 - e^{-kt}) - g/k\, t + H}$

Functional Form



initial slope = 0

final slope = $-g/k$ = terminal speed