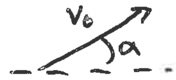
BALLISTIC TRAJECTORY FUN



$V_{0x} = V_0 \cos(\alpha)$

$V_{0y} = V_0 \sin(\alpha)$

Velocity

$V_x = V_0 \cos(\alpha)$; $v_y = v_0 \sin(\alpha) - gt$; $\vec{V} = \hat{\imath}\, v_x + \hat{\jmath}\, v_y = \hat{\imath}\, v_0 \cos(\alpha) + \hat{\jmath}\,(v_0 \sin\alpha - gt)$

Position

$\Delta x = x - x_0 = v_x t = v_0 \cos(\alpha)\, t$

$\Delta y = y - y_0 = v_{0y} t - \frac{1}{2} g t^2 = v_0 \sin(\alpha) t - \frac{1}{2} g t^2$

Acceleration

$a_x = 0$; $a_y = -g$; $\vec{a} = -g\hat{\jmath}$

Speed

$$v = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 \cos^2(\alpha) + v_0^2 \sin^2(\alpha) - 2v_0 \sin(\alpha) g t + g^2 t^2}$$

$$v = \sqrt{v_0^2 - 2v_0 \sin(\alpha) g t + g^2 t^2}$$

Special case: $\vec{V}_0$ pure upward, $\alpha = 90° \Rightarrow \sin(\alpha) = 1$

$$v = \sqrt{v_0^2 - 2v_0 g t + g^2 t^2} = \sqrt{(v_0 - gt)^2} = |v_0 - gt| \checkmark$$

Special case: $\vec{V}_0$ pure downward, $\alpha = -90° \Rightarrow \sin(\alpha) = -1$

$$v = \sqrt{v_0^2 + 2v_0 g t + g^2 t^2} = \sqrt{(v_0 + gt)^2} = |v_0 + gt| \checkmark$$

Rate of change of speed

$$\frac{dv}{dt} = \frac{d}{dt}\sqrt{v_x^2 + v_y^2} = \frac{d}{dt}(v_x^2 + v_y^2)^{1/2} = \frac{1}{2}(v_x^2 + v_y^2)^{-\frac{1}{2}}\left[\frac{d}{dt}(v_x^2) + \frac{d}{dt}(v_y)^2\right]$$

$$= \frac{1}{2v}\left(2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt}\right) = \frac{1}{2v}\left[0 + 2v_y(-g)\right] = -g\frac{v_y}{v}$$

This reduces to $-g$ when $v = v_y$; it can be +ve, −ve, or zero

Height dependence of speed

$\Delta y = v_0 \sin(\alpha) t - \frac{1}{2} g t^2$ so $2g\Delta y = 2v_0 \sin(\alpha) g t - g^2 t^2$

$$v = \sqrt{v_0^2 - 2v_0 \sin(\alpha) g t - g^2 t^2} = \sqrt{v_0^2 - 2g\Delta y}$$

no α-dependence!

Note also

$$v^2 = v_0^2 - 2g\Delta y$$

$v^2 - v_0^2 = -2g\Delta y$ even though v_x is nonzero!