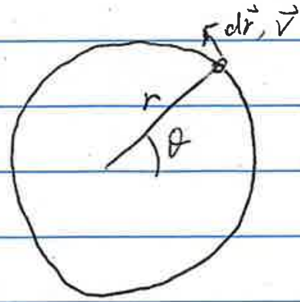
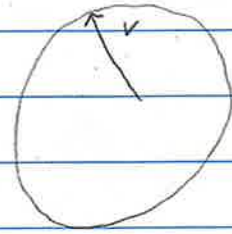
Kinematic Equations of Uniform Circular Motion by simple geometry



$v = 2\pi r / T$ = tangential speed



$a = 2\pi v / T$ T is period: time of 1 revolution

= rate of change of tangential speed

$a = 2\pi \, (2\pi r / T) / T$

$a = 4\pi^2 r / T^2 = \frac{2\pi^2 r^2}{T^2} / r = v^2 / r$

Directions: $\vec{v} \perp \vec{r}$, $\vec{a} \perp \vec{v}$ each leading by $90°$ ↖

(depending on direction of rotation)

$\vec{a}$ is inward no matter what the direction of travel

always toward the center: centripetal

We have already found two ways to express a:

$a = 4\pi^2 r / T^2 = v^2 / r$

We will also find it useful mathematically to use $\omega = d\theta/dt$

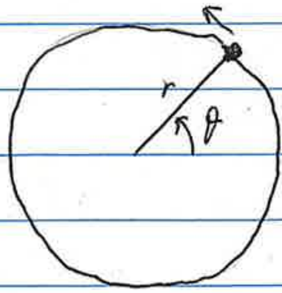
$\omega = d\theta/dt = 2\pi / T$

$a = (2\pi/T)^2 r = \omega^2 r$ $\quad$ $v = (2\pi/T) r = \omega r$

So we have $v = 2\pi r / T = \omega r$

$a = 4\pi^2 r / T^2 = \omega^2 r = v^2 / r$

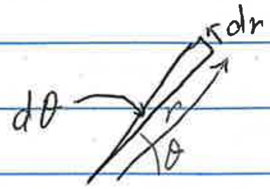
Equations of Kinematics of Uniform Circular Motion - by Geometry



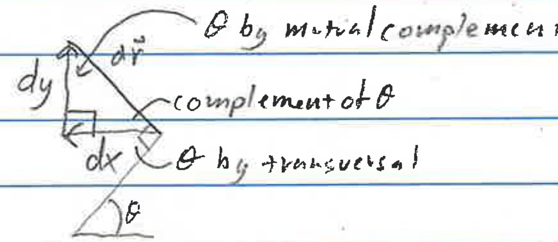
$\theta = \omega t + \phi$ $\qquad d\theta/dt = \omega$

$x = r_x = r\cos\theta$ $\qquad y = r_y = r\sin\theta$

$\vec{r} = r\cos(\omega t+\phi)\,\hat{\imath} + r\sin(\omega t+\phi)\,\hat{\jmath}$



$dr = r\,d\theta$



$dx = -\sin\theta\,dr$ $\qquad dy = +\cos\theta\,dr$

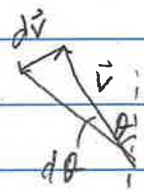
$\quad = -r\sin\theta\,d\theta$ $\qquad = +r\cos\theta\,d\theta$

$v_x = dx/dt = -r\sin\theta(d\theta/dt)$ $\qquad v_y = dy/dt = +r\cos\theta(d\theta/dt)$

$v_x = -r\sin\theta\,\omega = -\omega r\sin(\omega t+\phi)$ $\qquad v_y = +\omega r\cos(\omega t+\phi)$

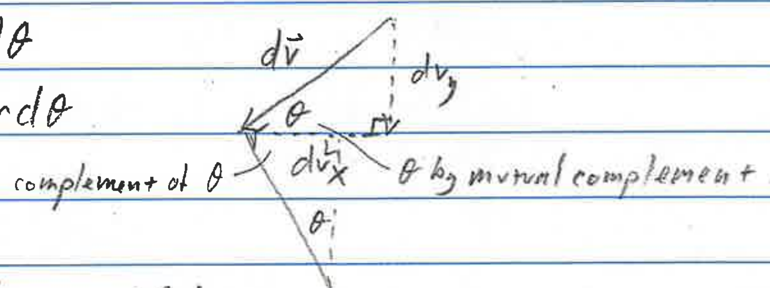
$\vec{v} = -\omega r\sin(\omega t+\phi) + \omega r\cos(\omega t+\phi)$

$v = \|d\vec{r}/dt\| = r\,d\theta/dt = r\omega$



$dv = v\,d\theta$

$dv = \omega r\,d\theta$



$dv_y = -\sin\theta\,dv$

$a_y = dv_y/dt$ $\qquad dv_x = -\cos\theta\,dv$

$\quad = -\sin\theta\,dv/dt$ $\qquad a_x = dv_x/dt = -\cos\theta\,dv/dt$

$\quad = -\sin\theta\,\omega r\,d\theta/dt$ $\qquad = -\cos\theta\,\omega r\,d\theta/dt$

$\quad = -\sin\theta\,\omega r\omega$ $\qquad = -\cos\theta\,\omega r\omega$

$a_y = -\omega^2 r\sin(\omega t+\phi)$ $\qquad a_x = -\omega^2 r\cos(\omega t+\phi)$

$\vec{a} = -\omega^2 r\cos(\omega t+\phi)\,\hat{\imath} - \omega^2 r\sin(\omega t+\phi)\,\hat{\jmath}$

$a = \|\vec{a}\| = dv/dt = \omega r\,d\theta/dt = \omega^2 r$