PHYS 1220-02 Group Work Sheet
Entropy and Equilibrium

1. A Carnot engine operates at the thermodynamic limit of efficiency for a heat engine. But no real machine anywhere is powered by a Carnot engine. For some reason, a Carnot engine isn't practical.

   The four steps of a Carnot cycle are 1. Isothermal expansion at T_H 2. Adiabatic expansion from T_H to T_C 3. Isothermal compression at T_C 4. Adiabatic compression from T_C to T_H. Which step or steps cannot be run optimally efficiently in practice, and why?

2. (From Levenspiel, *Understanding Engineering Thermo*, p. 237.) Close to ZigZag, Oregon, by the side of a dormant volcano, is a large 1-km^3 underground field of hot fractured quartz-like rock (density 2650 kg/m^3, c_p = 800 J/(kg K)). The average temperature of the rock is 500 K, of the surroundings is 280 K. I wonder how much useful energy is stored in that rock.

   a. How much work could be extracted from this rock by a heat engine running most efficiently? (Take into account that the temperature drops as heat is extracted.)

b. If we can sell electricity to the power company at 15 ¢/kW·h, how much is this energy source worth?

c. How long would the energy source last if work were extracted at the rate of 1.0 GW, the output of a large coal-fired power station?

3. Imagine a brass artifact of mass m_1 initially at temperature T_{1i} placed into a beaker containing mass m_2 of water initially at temperature T_{2i}. As heat flows away for the brass, its entropy decreases; as heat flows into the water, its entropy increases.

   a. The entropy change from a reversible heat flow is $dS = dq/T$. When a substance isn't changing its phase, a heat input causes a temperature increase, $dq = mcdT$. The specific heat c is a property of the material. Find the formula for the entropy change resulting from an input of heat Q to a sample of material with mass m, specific heat c, and initial temperature T_0.

   b. When the brass is immersed in the water, heat Q flows from the brass to the water. Find the formula for the change in entropy of the universe as a function of Q.

c. Now that you have a formula for the entropy change of the universe as a function of Q, the heat exchanged between the two items in contact with each other, find the formula for Q that maximizes entropy S. This is a max-min problem; you'll need to take the derivative of S with respect to Q dS/dQ, and find the Q at which this derivative is zero.

d. Now that you have found the heat exchange that maximizes entropy, find the formulas for

   i. The final temperature of object 1 (the initially hot one)

   ii. The final temperature of object 2 (the initially cold one).

e. Suppose for this situation that m_1 = 100.0 g, T_{1i} = 93°C, m_2 = 75.0 g, T_{2i} = 25°C. The specific heat of brass is c_1 = 402 J/(kg·K) and the specific heat of water is c_2 = 4184 J/(kg·K).

   i. What is the final temperature of the brass and water in thermal equilibrium?

   ii. What is the entropy increase of the universe as a result of this thermal equilibration?