LAB 10. TORQUE AND MOMENT OF INERTIA

Introduction

In this activity you will use a falling mass to pull a string, which will generate a torque to accelerate a heavy rotor with an unknown moment of inertia. You will measure the acceleration of the falling mass to determine the rotor's moment of inertia.

Apparatus

The rotor is mounted in the bearing with its axis vertical. The rotor is attached to three pulleys of different radius. One end of a string is wound around one of the pulleys, so that drawing the string horizontally away from the pulley produces a torque about the axis. The string is placed over a pulley and the other end is attached to a hanging mass, so that the weight of the hanging mass pulls on the string and the string in turn torques the rotor.

The pulley supporting the hanging mass is equipped with a photogate sensor, allowing the PASCO Data Studio software to measure the speed of the string running over it.

Theory

As the hanging mass falls, it turns the pulley attached to the axle of the rotor. When the hanging mass falls a distance d, the pulley, of radius r, advances an angle $\theta = d/r$ radians. Correspondingly, when the mass falls at speed v, the pulley rotates at angular speed $\omega = v/r$; if the acceleration of the falling mass is a, the angular acceleration of the pulley is $\alpha = a/r$.

The only force promoting the descent of the mass is its gravitational attraction mg to the earth. Opposing the acceleration is its mass and the rotational inertia (moment of inertia I) of the rotor. The tension T in the string linking the rotor and the hanging mass determines their acceleration: the net force on the hanging mass is a downward $\Sigma F = mg - T$, and the only torque on the pulley of the rotor is $\Sigma\tau = rT$. The acceleration of the hanging mass is then $a = \Sigma F/m$, and the angular acceleration of the rotor is $\alpha = \Sigma\tau/I$.

Experiment

In this activity you will pull on the string with three different hanging masses, and wind the string around the three different pulleys on the rotor axle. In each of the nine cases, you will measure the acceleration of the hanging mass.

Supplies

Rotor apparatus, string, photogate pulley, PASCO interface and computer with Data Studio software installed, 100-g, 200-g, and 500-g hanging masses, Vernier calipers, ruler.

Data Collection

Setup

1. Measure the mass and radius of the rotor.
2. Use the Vernier calipers to measure the diameters of each of the three pulleys on the rotor axle.
3. Connect the photogate pulley to the PASCO interface and recognize the sensor in Data Studio.

Measurement

1. Wind the string around one of the pulleys, leaving about 20 cm free.
2. Install the rotor into the bearing. Run the string over the photogate pulley and hang a mass at the end of the string.
3. Start data collection.
4. When the falling mass reaches the floor, stop data collection and stop the spinning rotor.
5. Make a velocity-time plot of the data in Data Studio. Fit the linear portion of the plot with a linear trend line. The slope of this trend line is the acceleration of the falling mass.
6. Record the hanging mass, the pulley radius, and the acceleration.
7. Repeat each run. If the two accelerations are not within 5% of each other, measure a third time.
8. Measure the acceleration of each of the three masses using each of the three pulleys on the rotor.

Data Processing

Theory

Solve the equations above to find the formula for acceleration in terms of m, r, and I.

Data Processing

1. Make a scatter plot of acceleration (vertical axis) vs. mr^2 (horizontal axis). Fit this with a one-parameter linear trend line, $y = Ax$. The slope of this trend line should be approximately g/I.
2. Calculate a first estimate of I from the slope A of the trend line.
3. Use this first estimate of I to calculate expected accelerations a_c using the formula determined earlier.
4. Calculate the "residual" σ, the difference between the predicted and observed accelerations, for each run using the formula $\sigma_i = a_{ci} - a_i$.
5. Make a plot of the residuals (vertical axis) vs. mr^2 (horizontal axis).
6. Calculate the "fit score" s as the sum of squared residuals, $s = \sum_{i=1}^{N} \sigma_i^2$.
7. If you haven't already set up a spreadsheet to automatically calculate the residuals σ_i and fit score s from a single value of I, do that now.
8. Experiment with different values of I to find the one giving the smallest s. This is the best-fit estimate of I.
9. Make another plot of the residuals (vertical axis) vs. mr^2 (horizontal axis) with this best-fit estimate.
10. Calculate with the moment of inertia should be for a uniform cylinder with the mass and radius of the rotor.

Lab Report

Present your findings in a brief, lucid report. It should contain the following parts.

Data

Show the raw data tables, reporting *m*, *r*, and *a* for each run.

Theory

Report your formula for *a* as a function of *m*, *r*, and *I*.

Results

Report the estimates of *I* from the first trend line, and the best-fit value. (Don't forget the units!) Also report the fit scores *s* associated with each estimate. Show the graphs that you made: *a* vs. mr^2 and its linear trend line, and the (two) residuals plots from the first estimate of *I* and the best-fit estimate.

Discussion

Do the predictions from the model using your best-fit estimate of *I* adequately match the observed accelerations? If yes, explain what constitutes "adequately". If no, suggest how the model could be modified to more faithfully fit the data.