LAB 12. HOOKE'S LAW

Introduction

External force applied to an object will change the object's size or shape or both. Whether the object springs back to its original form after the force is removed (elastic response) or remains deformed (plastic response) depends on the arrangement and bonding of the atoms in the material as well as the magnitude, rate and duration of force applied. The simplest approximation to the behavior of an elastic solid is Hooke's law, $F = -kx$, where F is the force exerted by the solid, k is the stiffness of the solid, and x is its distortion from its equilibrium size. Springs are constructed to display this behavior over fairly large extensions.

If a mass m is acted on only by a Hooke's law spring with force constant k, it oscillates with a period $T = 2\pi\sqrt{m/k}$. The angular frequency of the oscillation is $\omega = 2\pi/T = \sqrt{k/m}$.

In this lab you will use these two relations to obtain two different estimates of the stiffness k of several springs.

Theory

Extension

The extension x of a spring is its change in length: if its resting length is l_0 and its length under tension or compression is l, then $x = l - l_0$. Hooke's law $F = -kx$ tells the force F exerted by a spring as a function of the extension of the spring. In this lab, we will stretch a spring by hanging known masses from it. It will be natural for us to measure the static extension of the spring as a function of applied weight w, which by Newton's third law is $-F$. Then we can predict the length of the spring in response to the applied weight.

$$-w = F = -kx$$

$$w = kx = k(l - l_0) = kl - kl_0$$

$$kl = w + kl_0$$

$$l = w/k + l_0$$

Consequently, if Hooke's law is true, a plot of the length l of a spring vs. applied weight w should give a straight line with slope $1/k$ and y-intercept (actually l-intercept) l_0.

Oscillation

Likewise, we can measure the periods of oscillation T of different masses m on the spring. From T we can calculate the angular frequency ω. Squaring the formula for ω gives

$$\omega^2 = k/m$$

$$1/\omega^2 = m/k$$

This tells us that a plot of $1/\omega^2$ vs. m should give a straight line with slope $1/k$.

Equipment

Labeled springs, clamp stand, clamp, masses, stopwatch, meter stick.

Activities

Tension and extension

1. Measure and record the identifying number and initial length l_0 of a spring.
2. Hang a known mass from the spring and allow the mass and spring to come to rest. Record the spring's length l_1.
3. Find a set of at least seven (7) different masses that you can hang from the spring. They should all be heavy enough that they stretch the spring and light enough that they do not over-stretch it. Record the masses and the corresponding static spring lengths until you have seven (7) measurements $l_1 \ldots l_7$ in addition to the initial length l_0. Record these measurements.
4. Convert the masses to the forces they exert on the spring by multiplying by the gravitational field g. In other words, $w = mg$. Record these forces.
5. Repeat the process with a different spring.

Mass and Period

1. Record the identifying number of the spring.
2. Hang the spring from a rigid clamp or horizontal bar.
3. Find a set of at least five (5) different masses that you can hang from the spring. They should all be light enough that they do not over-stretch the spring, and heavy enough that they stretch the spring enough to clearly observe their oscillations. Record their masses.
4. Hang a mass from the spring. Raise and release the spring so that it oscillates. Ensure that it oscillates purely up and down; stop it and begin again if it swings from side to side. Also ensure that the mount remains fixed, and does not move with the oscillator.
5. Time a whole number of complete cycles of the oscillation. Use at least ten oscillations; more if the oscillation is rapid. Start the timer on "zero" and stop at the desired number of oscillations.
6. Stop the oscillations and start them again. Time the same number of oscillations. Repeat for three runs. Record the times and the number of oscillations.
7. Repeat the procedure for all of the different masses, three runs for each mass.
8. Repeat the process with the other spring.

Data

Tension and extension

Spring ______		Spring ______	
Hanging mass m	Length l	Hanging mass m	Length l
0		0	

Mass and Period

Mass m	Cycles N	Time NT			Mass m	Cycles N	Time NT		

Data Processing

Tension and extension

For each spring:

1. Plot a scatter graph of length (vertical axis) vs. load (horizontal axis).
2. Fit the scatter plot with a linear trend line $y = ax + b$. Record the parameters a and b.
3. Use the trend line equation to find the predicted value l_{calc} of spring length l for each load.
4. Calculate the residual $l_{\text{calc}} - l$ for each load.
5. Calculate the standard error σ for each load. Use the formula $\sigma = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}(l_{i\,calc} - l_i)^2}$.
6. Make a scatter plot of the residuals (vertical axis) vs. load (horizontal axis).

7. Calculate an estimate of the spring constant *k* from the slope parameter *a*.

Mass and Period

For each spring:

1. Divide the times by the number of oscillations to find an estimate of the period *T* for each run. Average these to find one *T* for each mass. Convert each *T* to an angular frequency ω.
2. Make a scatter plot of $1/\omega^2$ (vertical axis) vs. *m* (horizontal axis).
3. Fit the scatter plot with a linear trend line, $y = ax + b$. If you are using Google Sheets, select "Use equation" in "label." Record the trend line parameters *a* and *b*.
4. Use the trend line equation to calculate the *y* (that is, $1/\omega^2$) values corresponding to each of the *x* (that is, mass *m*) values used.
5. Calculate the oscillation period *T* corresponding to each calculated $1/\omega^2$.
4. Calculate the residual $T_{\text{calc}} - T$ for each load.
5. Make a scatter plot of the residuals (vertical axis) vs. mass (horizontal axis).
6. Calculate the estimate of *k* from the slope parameter *a*.
7. Note the intercept parameter. If the Hooke's law model applies, this should be close to zero.

Lab Report

Present your findings in a brief, lucid report. It should contain the following parts.

Tension and extension

Data

Show the raw data, reporting the number of the spring, the applied hanging masses, and the corresponding lengths of the spring.

Fitting

Show the graphs of *l* vs. *w* that you made. Include the trend lines.

Also show the residuals plots and report the values of the standard errors. (Don't neglect the units for the standard errors!)

Results

Report the equations for the trend lines and the corresponding estimates of the spring constants *k*.

Discussion

Compare the experimental plots of *l* vs *w* to the fitted trend line. Answer the following questions in complete sentences.

- What are the physical meanings of the fit parameters *a* and *b* in the trend line equation?
- How did you determine the estimates of *k*?

- Does the model appear to adequately describe the data? What is your evidence? Discuss the residuals plots in your explanation.

Mass and Period

Data

Show the raw data, reporting m, the number N of oscillations, and the time NT for each run. Also report the average period T for each spring/mass combination.

Fitting

Show the plots of $1/\omega^2$ vs. m. Include the trend lines.

Also show the residuals plots for each fit.

Results

Report the equations for the trend lines and the corresponding estimates of the spring constants k.

Discussion

Compare the experimental plots of $1/\omega^2$ vs. m data to the best-fit trend lines. Answer the following questions in complete sentences.

- How did you determine the estimates of k?
- Do your observations and analyses provide evidence for or against the model that $\omega = \sqrt{k/m}$? Discuss the residuals plots in your explanation.
- Compare the estimates of k from this activity to the estimates from the static tension activity. Do they agree? If they do agree, what does that mean? If they do not agree, what does that mean?