
LAB 15. ENTROPY

Problem

- If individual objects behave randomly and independently of each other, do collections of many of them behave in predictable ways?
- Why do matter and energy tend to disperse over time?

Equipment

poster board divided into five zones, ten cups, 100 dice, 2 rolling trays; large plugged funnel, BB's

Background

It is common in physics for many small objects, individually behaving randomly, to form a whole that acts quite predictably. In this activity we will explore the basis of the physical property known as *entropy*, the tendency of matter and energy to spread out rather than to accumulate in a particular location.

Entropy

All matter consists of microscopic particles that follow the same physical laws that govern larger objects: they accelerate in a gravitational field, any force they exert on another object is matched by an opposite force acting on them, and so on. We refer to these particles as molecules, although they could in fact be electrons, individual atoms, molecules in the chemical sense, or groups of molecules clumped together.

We will explore trends over time of two characteristics of molecules: location and kinetic energy. Location is just a matter of where the molecule is; kinetic energy depends on the speed of the molecule, without regard to the direction it is moving. If every molecule in a large sample has a small chance of moving away from its position, with motion in all directions equally likely, how does the large sample itself move about over the course of time? Likewise, when two molecules interact, if every molecule has an equal chance of either transferring some of its kinetic energy to the other molecule or of taking some kinetic energy from the other molecule, how will kinetic energy become distributed among the molecules?

Entropy is a technical term that quantitatively describes how evenly distributed some physical quantity is: high entropy means that molecules are spread out to fill all available space, and kinetic energy is evenly distributed so that molecules tend not to have very much more or less than the average. A critical principle in physics is that in a closed system, that is, one that neither receives energy from nor loses energy to its surroundings, *entropy increases as time progresses*. Sometimes this is achieved by spreading out molecules, and sometimes by spreading out energy. Sometimes both matter and energy become more spread out. Sometimes, however, one spreads out while the other becomes more localized. For that to occur, the system must gain more entropy from spreading out the one than it loses from localizing the other.

Activities

Make sure that you make all the needed measurements and finish all the activities during the lab period. The questions and analysis sections may be finished outside of the lab.

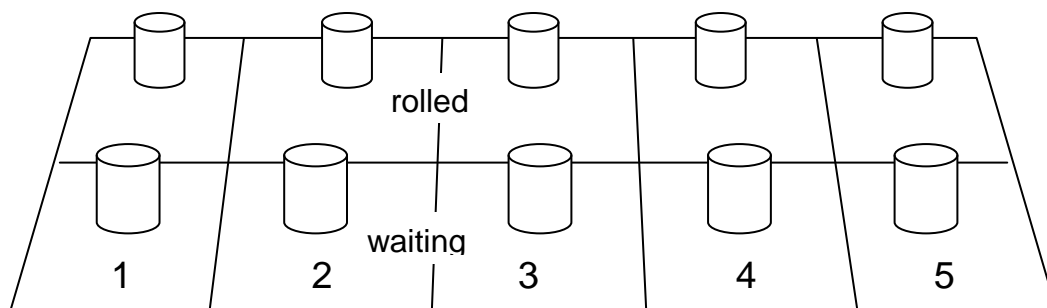
Activity 1: Going Places

In this activity, you will investigate how individual molecules move among the five zones of a game board when they have no preference for moving to the left or to the right. In a given step, a molecule may either move one space to the left, stay where it is, or move one space to the right. The molecules behave completely independently of each other; whether a molecule moves and which direction it moves are not affected by where any of the other molecules are or how they move.

Each molecule is represented by a six-sided die, and its movement is determined by the roll of the die. A roll of 1 means that the molecule moves one zone to the left, a roll of 2–5 means that the molecule stays in place, and a roll of 6 means that the molecule moves one zone to the right. If a molecule is already in the far-left zone (1), a roll of 1 means that it will move to the far-right zone (5); correspondingly, if the molecule is in the far-right zone (5), a roll of 6 means that it will move to Zone 1, the far-left zone.

Procedure:

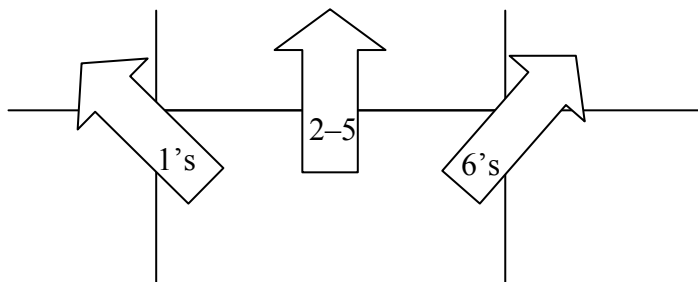
1. Place one cup in each of the ten regions of the board: “rolled” and “waiting” of each of zones 1–5.



2. Count out five (5) dice.
3. Put all of the dice into the “waiting” cup of Zone 3.
4. In Table 1, Step 0, record that all five dice are in Zone 3.

First Roll:

5. Gently roll the dice into the tray. The tray keeps them from rolling off the table, getting lost, or becoming mixed with another group's dice.



6. Remove all dice that come up “1” and put them into the “rolled” cup of the zone to the left. (That is Zone 2.)
7. Remove all dice that come up “6” and put them into the “rolled” cup of the zone to the right. (That is Zone 4.)
8. Remove all the remaining dice (rolled “2” through “5”) and put them into the “rolled” cup of Zone 3. These are the molecules that do not move in this step.
9. Record the number of dice in each zone at the conclusion of this step (1) in Table 1.
10. All the dice should be in the “rolled” cups in their zones. Switch each “rolled” cup with the “waiting” cup in the same zone so that all dice are now in the “waiting” cups.

Second and Later Rolls:

11. If there are any dice in the “waiting” cup for Zone 1, roll them. Place the rolled dice into the “rolled” cup of the zone where their numbers direct them to go: 1’s go to Zone 5, 2’s–5’s remain in Zone 1, and 6’s go to Zone 2.
12. Now, if there are any dice in the “waiting” cup for Zone 2, roll them. Place dice coming up “1” into the Zone 1 “rolled” cup, dice coming up “2”, “3”, “4”, or “5” into the Zone 2 “rolled” cup, and dice coming up “6” into the Zone 3 “rolled” cup.
13. Carry out something like step 12 for Zones 3 and 4. Roll only the dice in the “waiting” cups, not in the “rolled” cups! Dice coming up “1” go into the “rolled” cup of the zone to the left, dice coming up “6” go into the “rolled” cup of the zone to the right, and all other dice go into the “rolled” cup of the zone where they were.
14. Finally, if there are any dice in the “waiting” cup for Zone 5, roll them. Dice coming up “1” go into the “rolled” cup of Zone 4; dice coming up “6” go to the “rolled” cup of Zone 1, and all other dice go into the “rolled” cup of Zone 5.
15. Count the number of dice in each zone and record in Table 1. Switch the “rolled” cup for the “waiting” cup in each zone.
16. Repeat steps 11–15 until you have completed all 20 rows of Table 1.

Table 1. Distribution of 5 dice in 5 zones.

Step	Dice in Each Zone					Ω	$\ln \Omega$
	1	2	3	4	5		
0							
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							
19							
20							

17. Ω is the “multiplicity” of that arrangement of dice: the number of different ways the dice can be placed in the zones while maintaining n_1 dice in zone 1, n_2 dice in zone 2, and so on.

Complete the “ Ω ” column using the formula $\Omega = 5!/(n_1!n_2!n_3!n_4!n_5!)$. Here $N!$ means “ N factorial.” $N! = N \cdot (N-1) \cdot (N-2) \cdots 2 \cdot 1$. So, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. $1! = 1$, and by definition $0! = 1$ as well.

18. Complete the “ $\ln \Omega$ ” column. $\ln X$ means the “natural logarithm of X ,” when e is raised to the power of X , the result is $\ln(e^X) = X$. Useful values are $\ln(1) = 0$, $\ln(5) = 1.609$, $\ln(10) = 2.303$, $\ln(20) = 2.996$, $\ln(30) = 3.401$, $\ln(60) = 4.094$, and $\ln(120) = 4.787$.

Larger sample, better statistics:

19. Carry out steps 2–18 above, but this time use all one hundred (100) dice in step 2. Begin with all the dice in Zone 3 as in the previous run. Enter your data in Table 2. Most values of Ω will be too large to compute directly. Instead, use the on-line spreadsheet or other automated computation to calculate $\ln \Omega$.

Table 2. Distribution of 100 dice in 5 zones.

Step	Dice in Each Zone					$\ln \Omega$
	1	2	3	4	5	
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

Activity 2: Conflicting Tendencies

Sometimes, spreading out molecules and spreading out energy oppose each other. We will examine a simple example of this phenomenon and try to understand why the physical process goes in the direction it does.

The funnel you will use is plugged so that the bb's cannot come out of the spout. Hold the funnel upright (with its spout pointing downward and resting on the table, and its mouth at the top). Try to place the bb's at different positions in the mouth of the funnel. After positioning each bb, release it and watch where it goes.

1. What happens to the bb's? After releasing them, do they become more spread out or closer together?

Lab Report:

Complete the spreadsheet. Share it with me. Answer the questions below.

Your spreadsheet automatically makes four line graphs: the distributions of dice in the zones for both runs, and the value of $\ln(\Omega)$ in both runs.

1. Overall, how did $\ln \Omega$ change with time?
2. In either run, if $\ln \Omega$ ever decreased, what was the typical numerical value of the decrease?
3. In the "Conflicting Tendencies" activity, why do the bb's drop to the bottom of the funnel and stay there? Lest you be tempted to answer that things fall downward, not upward, remember that when things land, they usually bounce back upward. Why do they eventually stop bouncing?