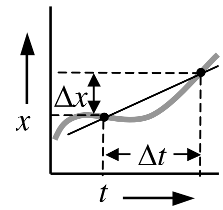
Worksheet 1: One-Dimensional Kinematics

Objectives

- Relate position, velocity, and acceleration in examples of motion along one dimension.
- Visualize motion using graphs of position, velocity, and acceleration vs. time.
- Solve numeric problems involving constant velocity and constant acceleration.

Summary

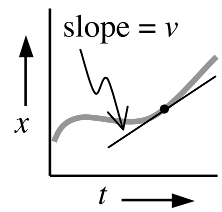


Average velocity is the ratio of how far an object moves to the time elapsed.

$$v_{avg} = \Delta x/\Delta t$$

On a plot of position vs. time, v_{avg} is the slope of the secant line connecting the starting and ending events.

Instantaneous velocity is the limit of average velocity as the time interval becomes infinitesimally brief. It is the velocity at a particular instant of time.



$$v = \lim_{\Delta t \to 0} \Delta x/\Delta t = dx/dt$$

On a of position-time plot, v is the slope of the tangent line at that particular instant. Conversely, the area under a velocity-time plot is the change in position, $\int_0^t v\,dt = \Delta x$.

Average acceleration is the ratio of the change in an object's velocity to the time elapsed.

$$a_{avg} = \Delta v/\Delta t$$

On a plot of velocity vs. time, a_{avg} is the slope of the secant line connecting the starting and ending events.

Instantaneous acceleration is the limit of average acceleration as the time interval becomes infinitesimally brief. It is the acceleration at a particular instant of time.

$$a = \lim_{\Delta t \to 0} \Delta v/\Delta t = dv/dt = d^2x/dt^2 ; \qquad \int_0^t a\,dt = \Delta v$$

On a of velocity-time plot, a is the slope of the tangent line at that particular instant. Conversely, the area under an acceleration-time plot is the change in velocity.

Kinematic Formulas

When velocity is constant, $\Delta x/\Delta t$ is the same for any time interval. Then $x = x_0 + vt$.

When acceleration is constant, $v = v_0 + at$; $x = x_0 + v_0 t + \frac{1}{2}at^2$.

Algebraic substitution allows us to find relations not requiring t or not requiring a:

$$2a(x - x_0) = v^2 - {v_0}^2; \qquad x - x_0 = \frac{v_0 + v}{2}t$$

Problems

There is not sufficient space on this worksheet to work these problems. Use plenty of scratch paper to work them.

Always follow this general strategy when working physics problems:

1. Consider the situation. Identify all relevant known and unknown physical quantities.
2. Identify the physics principles that apply to the situation. Describe a general approach to the problem.
3. Make a neat diagram of the situation. Include labeled coordinate axes and a table of numeric values (with units) of all known quantities.
4. Write down all equations that are relevant to the situation. Use only symbols for the quantities.
5. *Symbolically* solve the equation(s) for the desired quantity.
6. Verify your formula. Does it give the correct units? Are the relations between quantities reasonable? Does it satisfy the original equation?
7. Substitute the numeric values into the formula.
8. Check your answer. Does it satisfy your formula? Is it reasonable?

1. Earthquakes produce several types of shock waves. The most well-known are the P-waves (P for primary) and the S-waves (S for secondary). In the earth's crust, the P-waves travel at around 6.5 km/s, while the S-waves travel at about 3.5 km/s. The time delay between the arrival of these two waves at a seismic recording station tells geologists how far away the earthquake occurred.

   a. Determine a formula telling distance between the waves' origin and the detector from time delay between the arrival of the P- and S-waves at the detector.

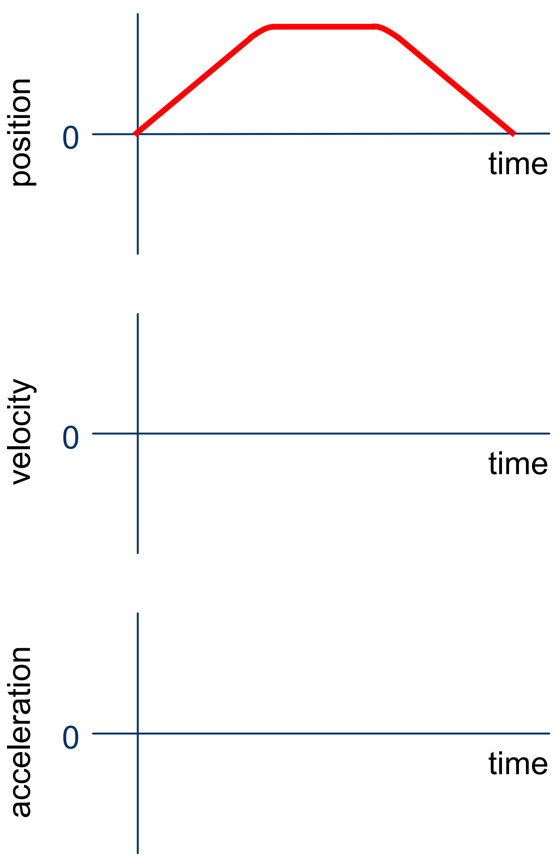
   b. If the time delay is 33 s, how far away from the seismic station did the earthquake occur?

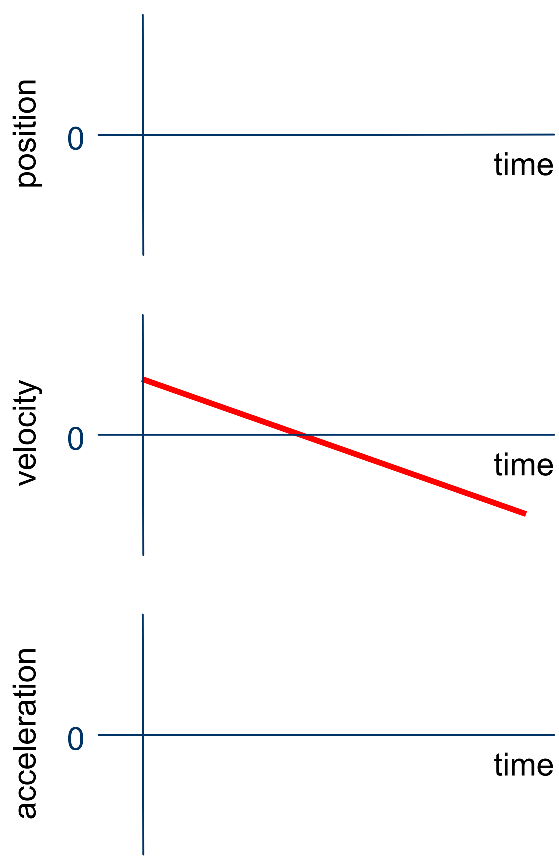
2. On a 20-mile bike ride, you ride the first 10 miles at an average speed of 8 mi/h.

   a. What must your average speed over the next 10 miles be to have your average speed for the total 20 miles be 4 mi/h?

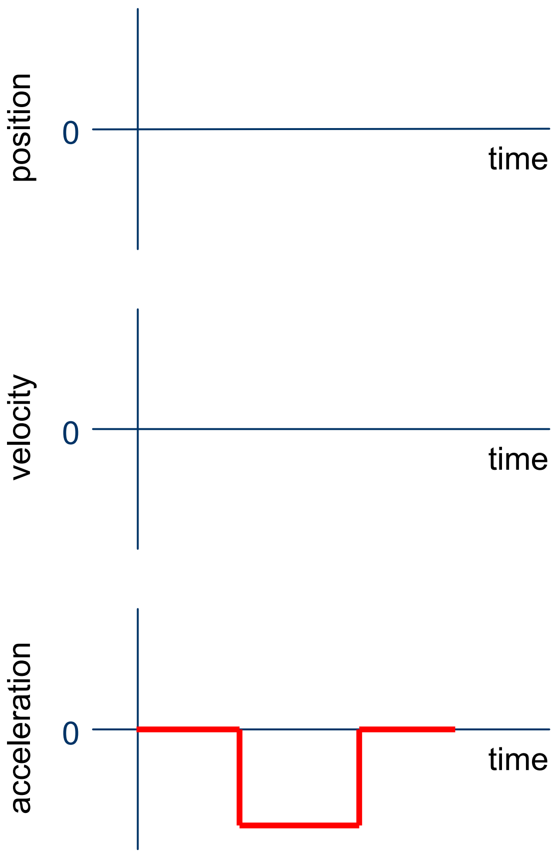
   b. Given your average speed for the first 10 miles, can you possibly attain an average speed of 16 mi/h for the total 20-mile ride?

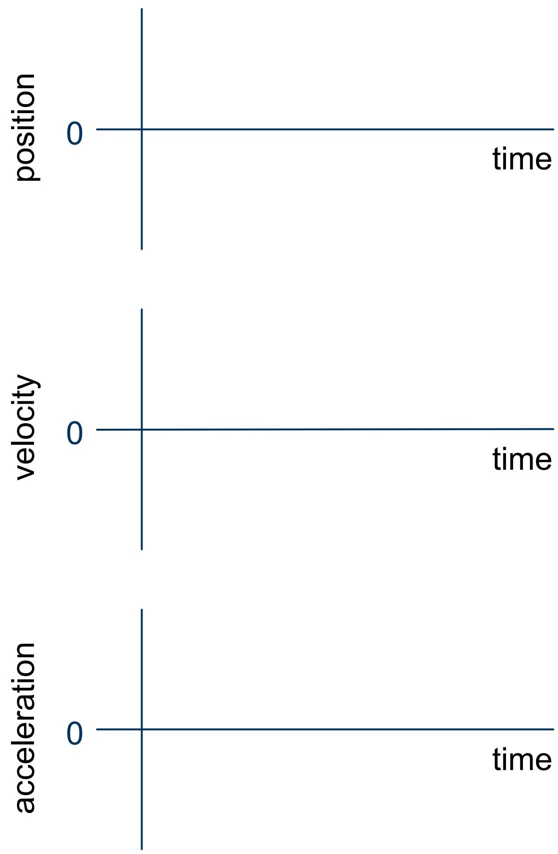
3. In the following scenarios, the motion of an object is to be described in four ways: (i) in words, (ii) as a position-time graph, (iii) as a velocity-time graph, and (iv) as an acceleration-time graph. In each case, only one description is given. Construct the other three. (You may need to assume some initial conditions.)





A coconut hangs motionless from its tree, then drops with increasing downward speed until it lands on the ground, quickly coming to rest.





4. A ball starts from rest and rolls down an incline at a constant acceleration. In 5 s, it rolls a distance of 50 m down the hill.

a. What is its acceleration?

b. If the same ball rolls down the same incline with the same acceleration, but begins with an initial downhill velocity of 2.0 m/s, how far down the hill will it be in 5 s?

c. If the ball begins with an initial uphill velocity of 2.0 m/s, where will it be in 5 s?

5. The human body can survive an acceleration trauma incident (sudden stop) if the magnitude of the acceleration is less than 250 m/s^2. If you are in an automobile accident with an initial speed of 30 m/s and you are stopped by an airbag that inflates from the dashboard, over what distance must you stop for you to survive the crash?

6. An automobile accelerates constantly from rest, traveling 400 m in 20.0 s.

a. What is its average velocity over this interval?

b. What is its final velocity?

c. The car's engine is adjusted, and the car is again accelerated constantly from rest through a distance of 400 m. This time, its final velocity is 50 m/s. What is the time elapsed?

7. A driver in Massachusetts was sent to traffic court for speeding. The evidence against the driver was that a policewoman observed the driver's car alongside a second car at a certain moment, and the policewoman had already clocked the second car as going faster than the speed limit. The driver argued, "The second car was passing me. I was not speeding." The judge ruled against the driver because, in the judge's words, "If two cars were side by side, you were both speeding." If you were a lawyer representing the accused driver, how would you argue this case?

8. The driver of a car wishes to pass a truck that is traveling at a constant speed of 20.0 m/s. Initially, the car is also traveling at 20.0 m/s and its front bumper is 24.0 m behind the truck's rear bumper. The car accelerates at a constant 0.600 m/s^2, then pulls back into the truck's lane when the rear of the car is 26.0 m ahead of the front of the truck. The car is 4.5 m long and the truck is 21.0 m long.

a. How much time is required for the car to pass the truck?

b. Check your answer by substituting it into your formula.

c. What is the final speed of the car?

9. One of your kinematic formulas is $x - x_0 = \frac{v + v_0}{2} t$.

a. Under what conditions does this formula apply?

b. Rearrange this formula to give an expression for the average velocity $v_{avg} = (x - x_0)/(t - t_0)$ under those conditions.

c. Under conditions of constant acceleration, instantaneous velocity at time t is given by $v = v_0 + at$. Substitute this expression for v into the kinematic formula and solve for x. Does this result make sense?

10. Another kinematic formula is $2a(x - x_0) = v^2 - v_0^2$.

a. Solve this formula for velocity v.

b. Check the units.

c. Under conditions of constant acceleration, position x is given by $x = x_0 + v_0 t + \frac{1}{2}at^2$. Substitute this expression for x into the expression you just found for v and simplify.

d. Does this result make sense?

11. Instantaneous velocity is the derivative of position with respect to time. Given the kinematic expression for position x at time t is $x = x_0 + v_0 t + \frac{1}{2}at^2$, find the velocity $v = dx/dt$.

12. A car 3.5 m in length traveling at 20 m/s approaches an intersection. The width of the intersection is 20 m. The light turns yellow when the front of the car is 50 m from the beginning of the intersection. The light will be yellow for 3 s.

a. If the driver steps on the brake, the car will slow at -3.8 m/s^2. Will the car stop before the intersection?

b. If the driver steps on the gas, the car will accelerate at 2.3 m/s^2. Will the car clear the intersection before the light turns red?

13. A student is running at her top speed of 5.0 m/s to catch a bus, which is stopped at the bus stop. When the student is still 40.0 m from the bus, it starts to pull away, moving with a constant acceleration of 0.170 m/s^2.

a. For how much time and for what distance must the student run before she overtakes the bus?

b. When she reaches the bus, how fast is the bus traveling?

c. Sketch an x-t graph for both the student and the bus.

d. The equations you used to find the time have a second solution, corresponding to a later time for which the student and bus are again at the same place if they continue their specified motion. Explain the significance of this second solution.

e. What is the minimum speed the student must have to just catch the bus?

f. If the student runs to just catch the bus, how far does she run to catch it?