LAB 20. KIRCHHOFF'S RULES

Problem

How can Kirchhoff's rules model voltages and currents in complex circuits?

Equipment

Digital multimeter with ohmmeter and voltmeter functions; five different resistors; wires (at least five); breadboard; alligator clips; one battery and one plug-in DC power supplies

Background

In previous exercises you learned how to build circuits from schematic diagrams and to measure resistance, current, and voltage in simple circuits. This week you will practice and strengthen these skills while investigating circuits whose behavior can only be modeled by using Kirchhoff's rules.

Kirchhoff's rules are two conservation equations expressed in terms of specific circuit characteristics. The loop rule or voltage rule is a consequence of conservation of energy. It states that the voltages across each component in a loop sum to zero.

$$0 = \Sigma V_i$$

The junction rule or current rule is a consequence of conservation of charge, specifically, that no point is a source or sink of electric charges. It states that the current originating at any point in a circuit is zero: any current coming out of a point is balanced by current coming into the same point.

$$0 = \Sigma I_i$$

Since one of these equations is in terms of voltages and the other in terms of current, it is necessary to be able to relate the two. For this, we use the familiar voltage-current relation for resistors

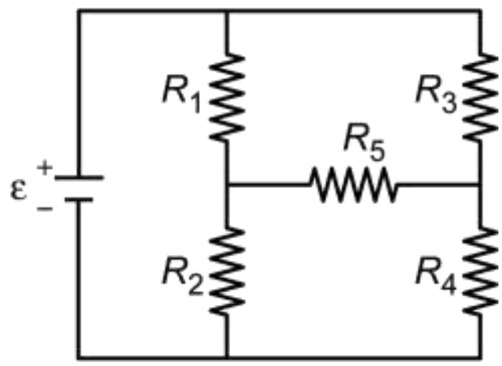
$$V = IR$$

where V is the potential drop across the resistor, I is the current through the resistor, and R is the resistance of the resistor. In both SI and common American usage, potential is measured in volts (V), current in amperes (A), and resistance in ohms (Ω).

Circuits

1. Bridge

The bridge circuit has the schematic structure below.



1. Select five resistors with different resistances. As a practical matter, you may wish to label each resistor so that you know which it is at a glance without working out its code. Stabbing one of its leads through a small scrap of paper with a number scrawled on it ought to suffice.
2. Using the multimeter in ohmmeter mode, individually measure the resistance of each resistor. Be sure to use the lowest setting that will give a reading in range. Record the resistance values.
3. Construct the circuit shown above. You may use any of your resistors in any position. List the resistor values for each position in the table below.
4. Decide on your convention for the direction of current through each resistor. For your own reference, draw them as arrows by each resistor in the diagram.
5. Using the multimeter in voltmeter mode, measure the voltage across the potential source (ε) and across each resistor. The sign of the voltage depends on which voltmeter probe you place on which side of a resistor. Use the convention that the (+) probe goes on the first (up-current) side encountered when moving in the direction of the current arrow, and the (–) probe on the second (down-current) side. Record the values below, making note of the signs.

ε = ____________________

R_1 = ____________________ V_1 = ____________________ I_1 = ____________________

R_2 = ____________________ V_2 = ____________________ I_2 = ____________________

R_3 = ____________________ V_3 = ____________________ I_3 = ____________________

R_4 = ____________________ V_4 = ____________________ I_4 = ____________________

R_5 = ____________________ V_5 = ____________________ I_5 = ____________________

6. Use $I = V/R$ to calculate the current I through each resistor. Enter in the table above.

2. Second Bridge

Switch at least two of the resistors in the circuit above. List the resistances, measure the voltages, and calculate the currents as before.

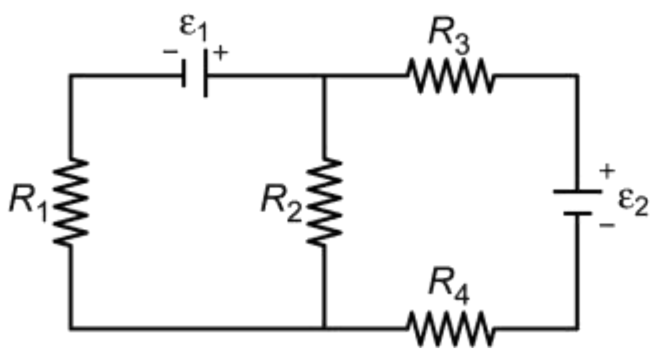
ε = ____________________

R_1 = ____________________	V_1 = ____________________	I_1 = ____________________
R_2 = ____________________	V_2 = ____________________	I_2 = ____________________
R_3 = ____________________	V_3 = ____________________	I_3 = ____________________
R_4 = ____________________	V_4 = ____________________	I_4 = ____________________
R_5 = ____________________	V_5 = ____________________	I_5 = ____________________

3. Two sources

This circuit uses only four resistors, but it makes up for it by having two voltage sources.

1. Using four of your resistors and two voltage sources, build the circuit below. You may put any resistor in any position.



2. Decide on your convention for current directions through each resistor. For your own reference, draw them as arrows by each resistor in the diagram.
3. Record all resistances, measure all voltages, and calculate all currents as before.

ε_1 = ____________________

ε_2 = ____________________

R_1 = ____________________	V_1 = ____________________	I_1 = ____________________
R_2 = ____________________	V_2 = ____________________	I_2 = ____________________
R_3 = ____________________	V_3 = ____________________	I_3 = ____________________
R_4 = ____________________	V_4 = ____________________	I_4 = ____________________

4. Flip a source

1. Keep the same circuit as before, but reverse the polarity of voltage source 2.
2. Draw all current arrows exactly the same as for the previous circuit.
3. Record all resistances, measure all voltages, and calculate all currents. (The sign of ε_2 will reverse from circuit 3.)

	ε_1 = ____________	
	ε_2 = ____________	
R_1 = ____________	V_1 = ____________	I_1 = ____________
R_2 = ____________	V_2 = ____________	I_2 = ____________
R_3 = ____________	V_3 = ____________	I_3 = ____________
R_4 = ____________	V_4 = ____________	I_4 = ____________

Data Transformations

Although you may have constructed the circuits and taken measurements as a group, each student should carry out the analysis individually.

First, use Kirchhoff's rules to make a set of independent linear equations for each circuit. Then enter the equations into a spreadsheet to model the circuits you investigated. Set up each spreadsheet page as shown in class: make a list of measured source voltages and resistor resistances, and incorporate these by reference into a coefficient matrix and constant column vector to express the equations. Invert the matrix to solve the simultaneous equations and thus find and display the column vector of the unknown currents. Calculate and display the voltages implied by the currents.

You need to set up only two sets of equations, because circuit 2 is the same as circuit 1 and circuit 4 is the same as circuit 3, differing only in specifics (resistances or source voltages). You can change the model from one of these pairs to the other by simply changing the constant vector in the spreadsheet. Once you are satisfied that your spreadsheet models a circuit correctly, make two copies, one for each of the two versions of the circuit.

Lab Report

Abstract

What did you physically do, what quantities did you measure, and what did you do with the measurements?

Purpose

Are you testing a hypothesis or verifying a claim? What insight should you gain from this activity?

Theory

In this lab, you use Ohm's law and Kirchhoff's circuit rules to model a real circuit. What are these laws, and what is their theoretical or empirical justification?

Experimental

Briefly describe how you constructed the circuits. Identify the components you used to construct them: batteries, voltage source, wires, breadboards, voltmeters, etc. A photograph of your circuit might be a nice touch.

Observations and data

Your data are recorded on this paper. Submit the four tables with your report.

Analysis and Discussion

The analysis is your set of equations. In this section, list the Kirchhoff's rules equations that are the elements of your set of simultaneous equations. Identify what part of the circuit each equation pertains to: left mesh, top junction between left and right mesh, etc. After this list of equations, write out the corresponding coefficient matrix, the constant matrix, and the unknown matrix that you gave to the spreadsheet to solve.

For the discussion, evaluate how closely the resistor voltages and currents calculated by the spreadsheet correspond to the voltages measured by your voltmeter and the currents calculated from the measured voltages and resistances. Suggest reasons for any substantive differences.

Conclusions

Do the equations from Kirchhoff's rules and Ohm's law realistically model the actual circuits? Does the analysis using the spreadsheet help you to understand the circuits?