LAB 28. IMPEDANCE

Problem

How are voltage and current relatedin RC an d LR alternating current circuits?

Equipment

Signal generator (computer with Capstone installed, ScienceWorkshop 750 Interface), connector wires, precise variable resistor, capacitors, inductors, AC multimeter

Background

Alternating Current RC Circuits

When a resistor and capacitor in series are driven by an alternating voltage of the form

$$v_S = V_S \cos(\omega t)$$

The voltages v_R across the resistor and v_C across the capacitor add so that $v_R + v_C = v_S$. The voltages v_R and v_C have the same frequency as the source, but different phases both from each other and from v_S. The current $i = i_S = i_R = i_C$ has the same phase as $v_R = iR$. The capacitor relation $C = q/v_C$ requires $i_C = dq/dt = C\frac{d}{dt}v_C$. The resulting specific formulas are

$$v_R = V_R \cos(\omega t + \varphi)$$

$$i = I \cos(\omega t + \varphi)$$

$$v_C = V_C \sin(\omega t + \varphi)$$

Here, $\tan \phi = V_C/V_R = \frac{1}{\omega CR}$, $V_R = V_S \cos \phi$, and $V_C = V_S \sin \phi$. Current i can be found from $I = V_R/R$. Alternatively, $I = \omega C V_C = V_C/X_C$, where X_C is the capacitive reactance $X_C = \frac{1}{\omega C}$.

We would like to have a generalization of resistance so that we can calculate I directly from the source voltage V_S. This is the impedance Z, which can be used $I = V_S/Z$. Its square is the sum of squares of the resistance and the capacitive reactance: $Z^2 = R^2 + X_C^2$.

Alternating Current RL Circuits

In this scenario, a resistor and inductor make a series circuit with the ac voltage source. Again, $v_S = V_S \cos(\omega t)$. The current $i = i_S = i_R = i_L$, and the current is in phase with the voltage across the resistor; $i = v_R/R$. The inductor relation $v_L = L di/dt$ allows us to predict the properties of the circuit. The specific formulas are

$$v_R = V_R \cos(\omega t + \varphi)$$

$$i = I \cos(\omega t + \varphi)$$

$$v_L = -V_L \sin(\omega t + \varphi)$$

Where $\tan\phi = -V_L/V_R = \frac{1}{\omega CR}$, $V_R = V_S\cos\phi$, and $V_L = -V_S\sin\phi$. Current i can be found from $I = V_R/R$. Alternatively, $I = V_L/(\omega L) = V_L/X_L$, where X_L is the inductive reactance $X_L = \omega L$. The impedance Z of an LR circuit is given by $Z^2 = R^2 + X_L^2$.

Activity

RC Circuit

1. Set the signal generator to produce sine wave output with an amplitude of $V = 1$ V. You may start with any source frequency f that you wish.
2. Place a capacitor in series with the variable resistor. Drive the series with a sine wave from the signal generator.
3. Check the voltage across the resistor V_R and the voltage across the capacitor V_C with the ac voltmeter. Adjust the resistor to make V_R and V_C similar to each other.
4. Measure the source voltage V_S, the voltage across the capacitor V_C, the voltage across the resistor V_R, and the current I through the circuit. Record the measurements, along with source frequency f and amplitude V, in your notebook.
5. Repeat at different frequencies. At each frequency, adjust the variable resistor to make V_R and V_C similar to each other. Record all measurements, frequencies, and amplitudes in your notebook.
6. Tag your capacitor so that you will be able to identify and use it in the future.

RL Circuit

1. Place an inductor in series with the variable resistor. Power the series with a sine wave from the signal generator.
2. At different source frequencies f, adjust the resistance to make the voltage across the resistor V_R and the voltage across the inductor V_L about the same. Record signal amplitude V, frequency f, measured voltage V_S, V_R, V_L, and current I in your notebook.
3. Tag your inductor so that you will be able to identify and use it in the future.

Data Processing

Calculations

Convert each frequency f to angular frequency ω. For the capacitors, calculate $\frac{V_R}{RV_C}$ or $\frac{V_R}{\omega RV_C}$ and plot against ω to visualize the capacitance at different frequencies. If it is constant, determine the capacitance.

For the inductors, calculate $\frac{RV_L}{V_R}$ or $\frac{RV_L}{\omega V_R}$ and plot against ω to visualize the inductance at different frequencies. If it is constant, determine the inductance.

Checks

Theoretically, for the capacitor $v_S^2 = v_R^2 + v_C^2$ and for the inductor $v_S^2 = v_R^2 + v_L^2$. Also theoretically, for the capacitive circuit $I = v_R/R = \omega C v_C$ and for the inductive circuit $I = v_R/R = v_L/(\omega L)$. It may be worth checking each of these to see how closely the components conform to their ideal formulas.