LAB 11. TORQUE AND MOMENT OF INERTIA

Introduction

In this activity you will use a falling mass to pull a string, which will generate a torque to accelerate a heavy rotor with an unknown moment of inertia. You will measure the acceleration of the falling mass to determine the rotor's moment of inertia.

Apparatus

The rotor is mounted in the bearing with its axis vertical. The axle of the rotor is equipped with three pulleys of different radius. One end of a string is wound around one of the pulleys, so that drawing the string horizontally away from the pulley produces a torque about the axis. The string is placed over a pulley and the other end is attached to a hanging mass, so that the weight of the hanging mass pulls on the string and the string in turn torques the rotor. An auxiliary mass can be attached to the rotor.

The pulley supporting the hanging mass is equipped with a photogate sensor, allowing the PASCO Data Studio software to measure the speed of the string running over it.

Theory

As the hanging mass falls, it turns the pulley attached to the axle of the rotor, so that the motion of the hanging mass and the rotor are related. When the hanging mass falls a distance d, the pulley, of radius r, advances an angle $\theta = d/r$ radians. Correspondingly, when the mass falls at speed v, the pulley rotates at angular speed $\omega = v/r$; if the acceleration of the falling mass is a, the angular acceleration of the pulley is $\alpha = a/r$.

The only force promoting the descent of the mass is its gravitational attraction mg to the earth. Opposing the acceleration is its mass and the rotational inertia I of the rotor. The tension T in the string linking the rotor and the hanging mass determines their acceleration: the net force on the hanging mass is a downward $\Sigma F = mg - T$, and the only torque on the pulley of the rotor is $\Sigma\tau = rT$. The acceleration of the hanging mass is then $a = \Sigma F/m$, and the angular acceleration of the rotor is $\alpha = \Sigma\tau/I$.

Experiment

In this activity you will pull on the string with three different hanging masses, and wind the string around the three different pulleys on the rotor axle. In each of the nine cases, you will measure the acceleration of the hanging mass. The experiments will be repeated with the auxiliary mass added to the rotor.

Supplies

Rotor apparatus, auxiliary mass (iron hoop), string, photogate "smart pulley", PASCO interface and computer with Data Studio software installed, 100-g, 200-g, and 500-g hanging masses, Vernier calipers, ruler.

Data Collection

Setup

1. Measure the mass and diameter of the rotor, and the mass and inner and outer diameters of the auxiliary mass.

2. Use the Vernier calipers to measure the diameters of each of the three pulleys on the rotor axle.
3. Connect the photogate pulley to the PASCO interface and recognize the smart pulley in Data Studio.

Measurements

1. Wind the string around one of the pulleys, leaving enough free to hang the pulling weight..
2. Install the rotor into the bearing. Run the string over the photogate pulley and hang a mass at the end of the string.
3. Start data collection.
4. Just before the falling mass reaches the floor, stop the rotor. Turn off data collection.
5. Make a velocity-time plot of the data in Data Studio. Fit the linear portion of the plot with a linear trend line. The slope of this trend line is the acceleration of the falling mass.
6. Record the hanging mass, the pulley radius, the fitted acceleration, and the uncertainty u (±) of the acceleration.
7. Repeat each run. If the two accelerations are not within 5% of each other, measure a third time.
8. Measure the acceleration of each of the three masses using each of the three pulleys on the rotor.
9. Repeat the entire procedure with the auxiliary mass added to the rotor.

Data Processing

Theory

Solve the equations above to find a formula for acceleration in terms of m, r, and I.

Data Processing

1. Make a scatter plot of acceleration (vertical axis) vs. mr^2 (horizontal axis). Fit this with a one-parameter linear trend line, $y = Ax$. (If mr^2 is small compared to I, the slope of this trend line should be approximately g/I.
2. Calculate a first estimate of I from the slope A of the trend line.
3. Use this estimate of I to calculate expected accelerations a_c using the formula determined earlier.
4. Calculate the "residual" σ, the difference between the predicted and observed accelerations, for each run using the formula $\sigma_i = a_{ci} - a_i$.
5. Calculate the "fit score" s as the sum of squared residuals: $s = \sum_{i=1}^{N} \sigma_i^2$.
6. Make a plot of the residuals (vertical axis) vs. mr^2 (horizontal axis).
7. If you haven't already set up a spreadsheet to automatically calculate the residuals σ_i and fit score s from a single value of I, do that now.

8. Experiment with different estimated values of *I* to find the one giving the smallest *s*. (Repeat steps 4–5 with different values of *I* to find the one giving the smallest *s*.) This is the best-fit estimate of *I*.
9. Make another plot of the residuals (vertical axis) vs. mr^2 (horizontal axis), using the residuals from the best-fit estimate.
10. Using the uncertainties *u* from the accelerations, calculate the expected fit score $\tilde{s} = \sum_{i=1}^{N} u_i^2$.
11. Repeat steps 1–9 for the rotor with the auxiliary mass attached.
12. Calculate what the moment of inertia of the apparatus should be. Approximate the rotor as a uniform cylinder, and the auxiliary mass as a hollow cylinder.

Lab Report

Present your findings in a brief, lucid report. It should contain the following parts.

Data

Show the raw data tables, reporting *m*, *r*, and *a* for each run.

Theory

Report your formula for *a* as a function of *m*, *r*, and *I*.

Report the calculated rotational inertias *I* for the rotor and for the auxiliary mass.

Results

For both the rotor alone and with the auxiliary mass added, report the estimate of *I* from the first trend line, and the best-fit *I* value. (Don't forget the units!) Also report the fit scores *s* and expected fit score $\tilde{s}$ associated with each estimate. Show the graphs that you made for each: *a* vs. mr^2 and its linear trend line, and the (two) residuals plots from the first estimate of *I* and the best-fit estimate.

Discussion

Compare the actual and expected fit scores. Do the predictions from the model using your best-fit estimate of *I* adequately match the observed accelerations? If yes, explain what constitutes "adequately". If no, suggest how the model could be modified to more faithfully fit the data.