LAB 13. SIMPLE HARMONIC MOTION

Introduction

The equation governing the motion of a mass m acted on only by a Hooke's law spring with force constant k is $m\, d^2x/dt^2 = -kx$. The general solution for displacement x of the mass as a function of time t is $x = A \cos(\omega t + \phi)$, where $\omega^2 = k/m$. In this lab you will test the claim that $\omega^2 = k/m$.

Theory

The quantities that you will measure are the mass m and the oscillation period T. T is related to the "angular frequency" ω by $\omega = 2\pi/T$. Substituting this into $\omega^2 = k/m$ and solving for T^2 yields

$$T^2 = \frac{4\pi^2}{k} m$$

Thus, a plot of T^2 vs. m should yield a straight line passing through the origin with a slope of $4\pi^2/k$.

Experiment

In this activity you will time the oscillations of different masses on the same spring. From your measurements, you will construct a plot of T^2 vs. m and find the least-squares best-fit line $y = Am$, where A is an adjustable parameter, for the data. Then you will solve the relation $A = 4\pi^2/k$ for k to estimate the force constant k of the spring. You will judge if the model $y = Am$ matches the data.

Supplies

Spring, clamp stand, clamp, masses, stopwatch.

Data Collection

Setup

1. Record the number of the label on the spring. (These are the same springs and numbers that we used in Lab 7.)
2. Hang the spring from a rigid clamp or horizontal bar.
3. Find a set of at least five (5) different masses that you can hang from the spring. They should all be light enough that they do not over-stretch the spring, and heavy enough that they stretch the spring enough to clearly observe their oscillations. Record their masses.

Measurements

1. Hang a mass from the spring. Raise and release the spring so that it oscillates. Ensure that it oscillates purely up and down; stop it and begin again if it swings from side to side. Also ensure that the mount remains fixed, and does not move with the oscillator.
2. Time a whole number of complete cycles of the oscillation. Use at least ten oscillations; more if the oscillation is rapid. Start the timer on "zero" and stop at the desired number of oscillations.
3. Stop the oscillations and start them again. Time the same number of oscillations. Repeat for three runs. Record the times and the number of oscillations.

4. Repeat the procedure for all of the different masses, three runs for each mass.

Follow-up

1. Find a Lab 7 report (yours or someone else's) that found the force constant k for your spring. Record whose report it is, and its estimate of k.

Data Processing

1. Divide the times by the number of oscillations to find an estimate of the period T for each run. Average these to find one T for each mass.
2. Make a scatter plot of T^2 (vertical axis) vs. m (horizontal axis).
3. Using the procedure presented in class to find the slope A, fit the scatter plot with a direct proportion trend line, $y = Am$.
4. Use this formula to calculate the y values corresponding to each of the masses m used.
5. Calculate the "residual" σ, the difference between the predicted and observed accelerations, for each run using the formula $\sigma_i = y_i - T_i^2$.
6. Make a plot of the residuals σ (vertical axis) vs. m (horizontal axis).
7. Calculate the estimate of k from the parameter A.

Lab Report

Present your findings in a brief, lucid report. It should contain the following parts.

Data

Show the raw data, reporting m, the number of oscillations, and the times for each run.

Fitting

Show the plot of T^2 vs. m. On the same axes, show the best-fit line $y = Am$.

Show the plot of residuals vs. m.

Results

Report the value of the parameter A, your corresponding estimate of the force constant k, and the estimate of k for the spring from the Lab 7 report. Cite the Lab 7 report.

Discussion

Compare the T^2 data to the best-fit model Am. Do your measurements provide evidence for or against this model? Comment on the residuals plot: does it show a pattern? Compare the estimate of k from this activity to the estimate from Lab 7.