PHYS 1110 Exam 1

Brief Solutions

1. Lab launcher

A. Components of initial velocity

The components of the vector $\vec{v}_0$ are the lengths of the legs of the right triangle which has $\vec{v}_0$ as the hypotenuse and the legs are parallel to the coordinate axes. Here, that gives $v_{0x} = v_0 \cos\theta = (4.00 \text{ m/s}) \cos 42° = 2.97$ m/s, and $v_{0y} = v_0 \sin\theta = (4.00 \text{ m/s}) \sin 42° = 2.68$ m/s.

B. Speed at the top

At the top of the trajectory, $v_y = 0$, but v_x is the same as always, $v_x = v_{0x}$. Thus the speed is slower than at launch.

C. Components of velocity at the top

We already identified the components to answer the previous question: $v_x = v_{0x} = 2.97$ m/s, and $v_{0y} = 0$.

D. Landing time from steeper launch

A steeper launch means that v_{0y} is greater. There are several ways to verify that the projectile will take longer to reach the ground. Perhaps the quickest mathematically is to first realize that $v_y^2 = v_{0y}^2 - 2g(y - y_0)$ requires $|v_y|$ to be greater at every height if v_{0y} is greater. On the way back down, then, v_y has changed more from $+v_{0y}$ to $-v_y$. Since v_y changes at a constant rate of g ($v_y = v_{0y} - gt$), that must require more time.

E. Speed at launch height

When the projectile returns to launch height, the vertical component of its velocity is the negative of its value at launch. The horizontal component of the velocity is constant throughout the trajectory. The speed at launch height is its launch speed, which is 4.00 m/s..

F. Direction at launch height

Although the projectile's speed at launch height is its launch speed, its direction is not the launch direction. Its direction is the negative of its launch direction, or $-42°$.

G. Range

The height of the projectile at any time is given by the equation $y = y_0 + v_0 t - 1/2\ gt^2$. Solving this equation for t when $y = 0$ tells when the projectile lands on the floor; using that time in the equation for horizontal component of position $x = x_0 + v_{0x}t$ will give the horizontal distance at landing, which is what we're looking for.

$$0 = y_0 + v_{0y}y - 1/2\ gt^2$$
$$0 = t^2 - 2\left(\frac{v_{0y}}{g}\right) - 2y_0/g$$

Then the quadratic formula tells us

$$t = \frac{v_{0y}}{g} \pm \sqrt{(v_{0y}/g)^2 + 2y_0/g}$$

Examination of the formula tells us that we want the larger solution, because the smaller solution is negative. The projectile was not undergoing projectile motion before time $t = 0$. The time we get is $0.26778 \text{ s} + \sqrt{(0.26778 \text{ s})^2 + (0.224 \text{ s})^2} = 0.8121$ s. Usong that time in the x equation gives $D = v_{0x}t = (2.972 \text{ m/s})(0.8121 \text{ s}) = 2.41$ m.

2. Force

Force is a vector quantity, with both magnitude and direction.

3. Net force

Net force is the total force: the vector sum of all forces applied to the body.

4. Newton in terms of kg, m, s

$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$

5. Coasting downhill

The elaborate description established that the net force on Ada was zero for the time described, so her acceleration was also zero. Newton's first law tells us that her velocity was constant throughout that time. At a constant velocity of 10 m/s, she will travel 50 meters in five seconds.

6. Earth-Sun gravity

The previous question asked about Newton's first law; this question asks about Newton's third law.

Newton's third law states that for every force, there is an equal and opposite "reaction" force. When object A applies a force to object B, then B applies a force to A of exactly the same magnitude and along the same line of action, but in the opposite direction. In the example of this question, the Sun exerts a force on the Earth, attracting the Earth to the Sun. The questions ask you to characterize the reaction to this force: the partner force in the interaction.

A. Body exerting the reaction force

The Earth.

B. Body receiving the reaction force

The Sun.

C. Direction of the reaction force

Toward Earth.

D. Strength of the reaction force

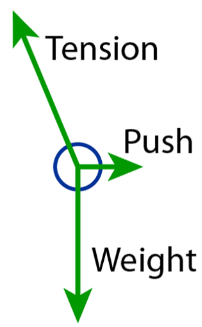
Same magnitude.

7. Safe hanging from a cable

There are three forces acting on the safe: its weight, magnitude mg, directed straight downward; the push form the worker, of unknown magnitude, directed horizontally to the right; and the pull of tension form the cable, directed upward to the left. This is a static situation, so all forces add to zero in both the vertical and horizontal directions.

A. Free body diagram

Depict the three forces as arrows outward from a circle representing the safe.



B. Weight of the safe

The safe's weight is its mass m multiplied by the local gravitational field g; $mg = (53.0\ \text{kg})(9.8\ \text{N/kg}) = 519.4\ \text{N}$.

C. Direction of tension force

Along the direction of the cable, inward. This is 16° left or counterclockwise of vertical, or 106° left or counterclockwise of the $+x$ axis, or 74° to the right, clockwise, or above the $-y$ axis.

D. Vertical component of the force exerted by the cable

This is calculated as a force of constraint: it must exactly cancel the other vertical forces on the safe. The only other vertical force is the safe's weight, downward at $mg = 519.4$ N, so the vertical component of tension must be upward at 519.4 N.

E. Horizontal component of the force exerted by the cable

This is also a force of constraint, exactly cancelling the worker's horizontal push. But here, we don't know the magnitude of the worker's push; only its direction.

We know the direction of the tension force, so we can determine the relative lengths of its components. We can depict the tension T and its x and y components T_x and T_y as the hypotenuse and catheti of a right triangle. The angle we know, $\theta = 16°$, is the upper angle, making T_x the opposite side and T_y the adjacent side. Thus the principal trigonometric ratios of the triangle are

$$\sin\theta = T_x/T$$
$$\cos\theta = T_y/T$$
$$\tan\theta = T_x/T_y$$

Here we are looking to find T_x. We know that $T_y = mg = 519.4$ N. The appropriate trigonometric function to use then is the tangent. Solving for T_x we find $T_x = T_y \tan\theta = mg \tan\theta = (519.4\ \text{N}) \tan 16° = 148.9\ \text{N}$.

8. Sliding into third base

A. Net force

We can find the net force if we know the runner's mass and acceleration. We need to use kinematics to find the acceleration. We know the runner's stopping distance $x - x_0 = 3.40$ m, initial speed $v_0 = 6.50$ m/s, and final speed $v = 0$. The most useful kinematic equation is $v^2 - v_0^2 = 2a(x - x_0)$;

to find acceleration we solve for a.

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (6.50 \text{ m/s})^2}{2(3.40 \text{ m})} = -6.213 \text{ m/s}^2$$

The negative sign means that acceleration is in the negative direction. Here's we've implied that forward is positive, so the runner's acceleration is backwards while he slides to a stop.

Once we know the acceleration, net force comes from $\Sigma F = ma = (44 \text{ kg})(-6.213 \text{ m/s}^2) = 273.4$ N.

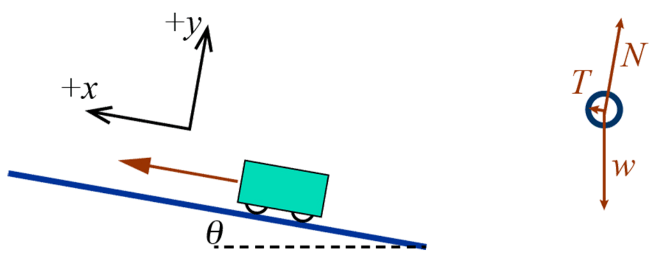
B. Normal force

The normal force is a force of constraint; it keeps objects from accelerating into surfaces. Here, it keeps the runner from accelerating downward, into the ground. The only force with a downward component is gravity, with magnitude mg; the normal force thus must be upward with magnitude $mg = 431.2$ N.

C. Coefficient of kinetic friction

We know the magnitude of kinetic friction from the net force. The net force is in the horizontal direction, and the only force on the runner in the horizontal direction is friction, so the net force must *be* friction, magnitude 273.4 N. Friction is proportional to the normal force, $f = \mu_k N$. The coefficient of kinetic friction μ_k thus is $\mu_k = f/N = (273.4 \text{ N})/(431.2 \text{ N}) = 0.63$.

9. Rescuing a car



In this problem there is an incline but no friction. The car will move along the incline, so it will be most convenient to use inclined coordinates, tilted by ten degrees. The components of the car's weight $mg = (1720 \text{ kg})(9.8 \text{ N/kg}) = 16856$ N are $w_x = -w \sin\theta = -2927$ N and $w_y = -w\cos\theta = -16600$ N.

A. Normal force

The normal force must exactly cancel the component of the car's weight perpendicular to the embankment, which is w_y. That's 16600 newtons.

B. Towing force

The towing force must give a barely uphill acceleration ($+x$ direction here), so it must be just larger than w_x of 2927 newtons.

10. One cart tows another

The greatest force that can pull the drive cart forward is friction between its wheels and the track, $f = \mu m_2 g$. The track is level, so the normal forces exactly cancel each car's weight. Tension in the cord between the carts pulls the pulled cart forward and the drive cart backward. Because the cord links the carts, their velocities and accelerations are the same.

For the pulled cart,

$$m_1 a = T$$

For the drive cart,

$$m_2 a = f - T$$

These are two equations in two unknowns, a and T. The unknown we are looking for is the acceleration a. We have the tools but not the motivation to find the tension T in the cord. We want to find a, which we will do by solving one of the equations for T in terms of a, substituting that into the other equation, and then solving that equation for a.

$$\begin{aligned}
m_2 a &= f - m_1 a \\
m_2 a &= \mu m_2 g - m_1 a \\
m_1 a + m_2 a &= \mu m_2 g \\
a(m_1 + m_2) &= \mu m_2 g \\
a &= \mu g \frac{m_2}{m_1 + m_2} \\
a &= 0.55(9.8\ \text{m/s}^2)\ \frac{0.200\ \text{kg}}{0.700\ \text{kg}} = 1.54\ \text{m/s}^2
\end{aligned}$$