PHYS 1110 Exam 2

Brief Solutions

1. Circling race car

A. Time of 1 revolution

Speed $v = \Delta s/\Delta t$; for one revolution, the path length $\Delta s = 2\pi r$. Thus $v = 2\pi r/T$, and time $T = 2\pi r/v = 2\pi(50.0\,\mathrm{m})/(15.0\,\mathrm{m/s}) = 20.94\,\mathrm{s}$.

B. Angular speed

Angular speed $\omega = v/r = (15.0\,\mathrm{m/s})/(50.0\,\mathrm{m}) = 0.30\,\mathrm{radians/s}$.

C. Direction of acceleration

The acceleration of a body in uniform circular motion id toward the center of its circular path.

D. Smallest practicable radius

The force of static friction provides the net force ΣF accelerating the car in its turn. Here we want the smallest radius turn, which means the greatest possible friction. The greatest possible value of static friction is $f_s = \mu_s N$; on level ground, $N = mg$. By Newton's second law, $a = \Sigma F/m$. In uniform circular motion,$a = v^2/r$, so

$$v^2/r = \mu_s mg/m$$

$$r = \frac{v^2}{\mu_s g} = \frac{(15.0\,\mathrm{m/s})^2}{0.70 \cdot 9.8\,\mathrm{m/s^2}} = 32.80\,\mathrm{m}$$

2. Dot and cross products.

A. Dot product and magnitudes

$\vec{A} \cdot \vec{B} = AB\cos\theta$; the dot product is directly proportional to the product of the magnitudes.

B. Cross product and magnitudes

$\|\vec{A} \times \vec{B}\| = AB\sin\theta$; the cross product is directly proportional to the product of the magnitudes.

C. Nature of dot product

Dot product is a scalar.

D. Nature of cross product

Cross product is a vector.

E. Dot product and angle between vectors

$\vec{A} \cdot \vec{B} = AB\cos\theta$; the dot product is directly proportional to the cosine of the angle.

F. Cross product and angle between vectors

$\vec{A} \times \vec{B} = AB\sin\theta$; the cross product is directly proportional to the sine of the angle.

3. Sled on a hill

A. Work done by gravity on the ascent

The force of gravity pulls straight down with magnitude mg. The work gravity does during a displacement depends only on the vertical component of the displacement. Here, the vertical component of Annie's displacement is $\Delta y = +4.0$ meters, so the work done by gravity is $-mg\Delta y = -(35.0\,\text{kg})(9.8\,\text{N/kg})(4.0\,\text{m}) = -1\,372\,\text{J}$.

B. Work done by normal force on the ascent

The normal force is perpendicular to Annie's displacement, so it does zero work.

C. Work done by friction on the ascent

Friction does negative work, always opposing the sliding. Its direction is directly opposite the direction of the displacement $\vec{s}$, so the work it does is $\vec{f} \cdot \vec{s} = -fs = -(2.00\,\text{N})(40.0\,\text{m}) = -80\,\text{J}$.

D. Work done by Barbara on the ascent

Annie's kinetic energy is zero at the bottom of the hill and at the top of the hill, so the work-energy theorem requires that the total work done on her is zero. The total work done is the sum of the contributions from gravity, friction, and Barbara's pull, so we can find W_B, the work done by Barbara, from the difference.

$$\begin{aligned} 0 &= -1\,372\,\text{J} + 0 - 80\,\text{J} + W_B \\ W_B &= 1\,372\,\text{J} + 80\,\text{J} \\ &= 1\,452\,\text{J} \end{aligned}$$

E. Barbara's power on the ascent

Power is the rate of doing work, so $P = W_B/\Delta t = (1\,452\,\text{J})/(200\,\text{s}) = 7.26\,\text{watts}$.

F. Net work done on the descent

On the descent, gravity does the opposite amount of work that it did on the ascent, because the force of gravity is the same direction as before, but now the displacement is the exact opposite of what it was before. The normal force is still perpendicular to the displacement, and again does zero work. Friction still opposes the sliding; the displacement vector reverses, and so does the friction vector, so the (negative) work done by friction is the same as on the ascent. Barbara does no work on Annie and the sled on their descent.

$$W = +1\,372\,\text{J} + 0 - 80\,\text{J} + 0 = 1\,292\,\text{J}.$$

G. Kinetic energy at the bottom of the hill

The work-energy theorem tells us that the work done equals the change in kinetic energy. Annie's initial kinetic energy was zero.

$$\begin{aligned} K_f - K_i &= W \\ K_f - 0 &= 1\,292\,\text{J} \\ K_f &= 1\,292\,\text{J} \end{aligned}$$

H. Speed at the bottom of the hill

Kinetic energy K depends on speeed as $K = 1/2\,mv^2$, so

$$v = \sqrt{2K/m} = \sqrt{\frac{2 \cdot 1\,292\,\text{J}}{35\,\text{kg}}} = 8.59\,\text{m/s}.$$

4. Snowman pinball

A. Compression length

Hooke's law tells us $F = kx$, where F is the force applied *to* the spring, k is the spring constant (stiffness), and x is the spring's distortion from its unstressed length. Here we are given F and k and asked to find x.

$$x = F/k = \frac{600\,\mathrm{N}}{1500\,\mathrm{N/m}} = 0.400\,\mathrm{m}$$

B. Spring potential energy

Potential energy of a Hooke's law spring is $U = 1/2\,kx^2 = 1/2\,(1500\,\mathrm{N/m})(0.400\,\mathrm{m})^2 = 120\,\mathrm{J}$.

5. Ballistic Pendulum

This is a two-step problem in which each step uses a different conservation principle. The bullet embedding into the block is a totally inerlastic collision in which total momentum is conserved. The upward swing of the block is a situation in which total mechanical energy is conserved, because the only force that does work on the block is gravity, a conservative force. (The tension in the cables does no work on the block, being always perpendicular to the block's path.)

A. Type of collision

Totally inelastic: the colliding bodies stick together.

B. Speed right after the collision

For a totally inelastic collision, we have

$$v_2 = \frac{m_B v_1 + m_W v_W}{m_B + m_W} = \frac{(0.010\,\mathrm{kg})(400\,\mathrm{m/s}) + (5.00\,\mathrm{kg})(0\,\mathrm{m/s})}{5.01\,\mathrm{kg}} = \frac{4.00}{5.01}\,\mathrm{m/s} = 0.798\,\mathrm{m/s}$$

C. Kinetic energy right after the collision

$$\begin{aligned}
K_2 &= 1/2\,(m_B + m_W)v_2^2 \\
&= 1/2\,(m_B + m_W)\left(v_1 \frac{m_B}{m_B + m_W}\right)^2 \\
&= 1/2\,m_B v_1^2 \left(\frac{m_B}{m_B + m_W}\right) \\
&= K_1 \left(\frac{m_B}{m_B + m_W}\right) \\
&= 1/2\,(0.010\,\mathrm{kg})(400\,\mathrm{m/s})^2 \left(\frac{0.01}{5.01}\right) \\
&= (800\,\mathrm{J})(1/501) \\
K_2 &= 1.597\,\mathrm{J}
\end{aligned}$$

D. Gravitational potential energy at the top of the swing

The swing step conserves total mechanical energy. Below I'll use $M = m_B + m_W$ to save space, because the bullet and block stay together and are essentially one thing. At the top of the swing, which I'll call state 3, the speed of the block is zero. The height y in the diagram is the increase in height between states 2 and 3. We can thus call the gravitational potential energy at the bottom zero, and at the top of the swing Mgy.

$$\begin{aligned} E_2 &= E_3 \\ 1/2\, M v_2^2 + 0 &= 0 + Mgy \\ 1/2\, M v_2^2 &= Mgy \end{aligned}$$

At the top of the swing, the potential energy of the block is what its kinetic energy was right after the collision.

E. Height at the top of the swing

We solved most of this problem in the previous step. Now all we need to do is solve for the height increase y.

$$\begin{aligned} 1/2\, M v_2^2 &= Mgy \\ 1/2\, v_2^2 &= gy \\ y &= \frac{v_2^2}{2g} \\ &= \frac{(0.798\,\mathrm{m/s})^2}{19.6\,\mathrm{m/s^2}} = 0.0325\,\mathrm{m} \\ &= 3.25\,\mathrm{cm} \end{aligned}$$

6. Elastic linear track collision

The pertinent values for this problem are

$m_1 = 1.00$ kg

$m_2 = 1.50$ kg

$v_{1i} = +1.20$ m/s

$v_{2i} = -0.300$ m/s

For an elastic collision in one dimension, our predictive formulas are

$$\begin{aligned} v_{1f} &= v_{1i}\frac{m_1 - m_2}{m_1 + m_2} + v_{2i}\frac{2m_2}{m_1 + m_2} \\ v_{2f} &= v_{2i}\frac{m_2 - m_1}{m_1 + m_2} + v_{1i}\frac{2m_1}{m_1 + m_2} \end{aligned}$$

A. Initial total momentum

$$\begin{aligned} \Sigma p = p_1 + p_2 &= m_1 v_{1i} + m_2 v_{2i} \\ &= (1.00\,\mathrm{kg})(1.20\,\mathrm{m/s}) + (1.50\,\mathrm{kg})(-0.300\,\mathrm{m/s}) \\ &= (1.20 - 0.45)\,\mathrm{kg\cdot m/s} \\ &= 0.75\,\mathrm{kg\cdot m/s} \end{aligned}$$

B. Final total momentum

Collisions conserve momentum, so the final total momentum is the same as the initial total momentum: $0.75\,\mathrm{kg\cdot m/s}$.

C. Final velocity of the 1-kg cart

For this and the next question, we just plug the values into the formulas above.

$$v_{1f} = \left(1.20\frac{-0.5}{2.5} + (-0.30)\frac{3.0}{2.5}\right)\,\mathrm{m/s} = -0.6\,\mathrm{m/s}$$

D. Final velocity of the 1.5-kg cart

$$v_{2f} = \left((-0.30)\frac{0.5}{2.5} + 1.2\frac{2}{2.5}\right)\,\mathrm{m/s} = +0.9\,\mathrm{m/s}$$

E. Momentum change of 1-kg cart

$$\Delta p = p_{1f} - p_{1i} = m_1 v_{1f} - m_1 v_{1i} = m_1(v_{1f} - v_{1i}) = (1.00\,\mathrm{kg})(-0.6 - 1.2)\,\mathrm{m/s} = -1.80\,\mathrm{kg}\cdot\mathrm{m/s}$$

F. Average interaction force

We use the impulse-momentum theorem to answer this question. That tells us that the change in momentum (which we found in the previous part) is equal to the impulse, which is force times the time during which it is applied, $J = F\Delta t$. Thus, $F = J/\Delta t = \Delta p/\Delta t$.

$$F = \frac{-1.80\,\mathrm{kg}\cdot\mathrm{m/s}}{0.50\,\mathrm{s}} = -3.60\,\mathrm{N}$$

Magnitude is non-negative, so the magnitude of the average force is 3.60 newtons.

7. Bicycle accelerating downhill

A. Initial angular speed

Angular speed $\omega = v/r = (3.00\,\mathrm{m/s})/(0.36\,\mathrm{m}) = 8.33\,\mathrm{rad/s}$.

B. Angular acceleration

Angular acceleration $\alpha = a_{\|}/r = (0.10\,\mathrm{m/s^2})/(0.36\,\mathrm{m}) = 0.278\,\mathrm{rad/s^2}$.

It turns out that the tangential and centripetal acceleration of a point on the rim of a rolling wheel are much more interesting (complicated) than I presented in class. The values I gave turn out to be just one term in the correct formulas. Both components of the acceleration additionally oscillate with each revolution of the wheel. This is because the acceleration of the center of mass of the wheel is sometimes in the radial direction, sometimes opposite the radial direction, sometimes in the tangential direction, sometimes opposite the tangential direction, and usually some combination of radial and tangential.

C. Centripetal acceleration

The answer I was looking for was v^2/r. It turns out that the value of the centripetal acceleration is actually v^2/r plus a term that oscillates between $\pm$ the center-of-mass acceleration with each revolution.

D. Tangential acceleration

The answer I was looking for was the center-of-mass acceleration. It turns out that the tangential acceleration is actually that value plus, again, a term that oscillates between $\pm$ the center-of-mass acceleration with each revolution, but 1/4 cycle out of phase with the oscillation in the centripetal acceleration.

Working on this problem was very enlightening for me.