PHYS 1110 Exam 3

Brief Solutions

1. Holding a bowling ball

A. Magnitude of torque from ball's weight

$$mgL\sin(90^\circ) = (6\,\mathrm{kg})(9.8\,\mathrm{N/kg})(0.34\,\mathrm{m})(1) = 20.0\,\mathrm{N\cdot m}$$

B. Direction of torque from ball's weight

Negative (clockwise).

C. Magnitude of torque from forearm's weight

$$mgC\sin(90^\circ) = (3\,\mathrm{kg})(9.8\,\mathrm{N/kg})(0.12\,\mathrm{m})(1) = 3.53\,\mathrm{N\cdot m}$$

D. Direction of torque from forearm's weight

Negative (clockwise).

E. Magnitude of torque from muscle

We don't know the force, so we need to use statics to find this torque. The net torque about the elbow must be zero. The torques extending the joint are from the weights of the forearm and of the bowling ball, which total $23.5\,\mathrm{N\cdot m}$. The torque flexing the elbow is from the biceps muscle, which must also be $23.5\,\mathrm{N\cdot m}$, in the opposite direction.

F. Direction of torque from muscle

The opposite direction is positive (counterclockwise).

G. Magnitude of tension in muscle

What force at distance $D = 0.0480\,\mathrm{m}$ gives a torque of $23.5\,\mathrm{N\cdot m}$? A force of 490 newtons.

H. Magnitude of torque from humerus

The force from the humerus is applied right at O, so it has no lever arm. Its torque is zero

I. Direction of torque from humerus

Zero.

J. Magnitude of force from humerus

The net force on the elbow must be zero. The other forces acting on the forearm are weights of $(9\,\mathrm{kg})(9.8\,\mathrm{N/kg}) = 88.2\,\mathrm{N}$ downward and a tension of 490 N upward, for a total of 401.8 N upward. The humerus thus must push downward at 401.8 N.

K. Direction of force from humerus

Downward.

2. Revving a rotor

The net torque on the rotor is the tension in the cord multiplied by the radius of the pulley. The angular acceleration of the rotor is its net torque divided by its moment of inertia.

$$\alpha = \Sigma\tau/I = Tr/I = (3.00\,\mathrm{N})(0.025\,\mathrm{m})/(0.0400\,\mathrm{kg\cdot m^2}) = 1.88\,\mathrm{rad/s^2}$$

3. Tumbling platform diver

The diver receives no external torques, so his angular momentum is conserved as he dives.

To start, the information we have about the diver is a mass of $M = 55\,\text{kg}$ and an initial angular velocity of $\omega_1 = 5.00\,\text{rad/s}$.

A. Moment of inertia in tuck

In his tuck, we are considering him a uniform solid sphere, which has a formula of $I = 2/5\,MR^2$. We are told his radius is $R = 0.25\,\text{m}$, so $I_1 = (0.4)(55\,\text{kg})(0.25\,\text{m})^2 = 1.375\,\text{kg}\cdot\text{m}^2$.

B. Moment of inertia extended

Here, we are told to consider him a uniform rod with length $L = 1.70\,\text{m}$, which gives us $I_2 = 1/12\,ML^2 = (1/12)(55\,\text{kg})(1.70\,\text{m})^2 = 13.26\,\text{kg}\cdot\text{m}^2$.

C. Angular momentum in tuck

Here we need to use the formula $l_1 = I_1\omega_1 = (1.375\,\text{kg}\cdot\text{m}^2)(5.00\,\text{rad/s}) = 6.875\,\text{kg}\cdot\text{m}^2/\text{s}$.

D. Angular momentum extended

We haven't been told his angular speed when extended, but we do know that his angular momentum is conserved. It must be the same as in the tuck, which was $6.875\,\text{kg}\cdot\text{m}^2/\text{s}$.

E. Angular velocity extended

We know his angular momentum l and moment of inertia I_2, so we solve $l = I\omega$ for ω, giving $\omega_2 = l/I_2 = (6.875\,\text{kg}\cdot\text{m}^2/\text{s})/(13.25\,\text{kg}\cdot\text{m}^2) = 0.52\,\text{rad/s}$.

F. Kinetic energy in tuck

$$K_{\text{rot}} = 1/2\,I\omega^2 = 1/2\,(1.375\,\text{kg}\cdot\text{m}^2)(5.00\,\text{rad/s})^2 = 17.2\,\text{J}$$

4. Weight oscillating on a spring

We know mass $m = 0.400\,\text{kg}$, amplitude $A = 0.085\,\text{m}$, and period $T = 0.80\,\text{s}$.

We'll want to use the angular frequency ω, which we'll get from the period via $\omega = 2\pi/T = (2\pi)/(0.80\,\text{s}) = 7.854\,\text{rad/s}$.

A. Spring constant

We find this from $\omega^2 = k/m$, because we know ω and m. Thus $k = \omega^2 m = (7.854\,\text{rad/s})^2(0.400\,\text{kg}) = 24.7\,\text{kg/s}^2 = 24.7\,\text{N/m}$.

B. Total energy

We know the maximum displacement of the spring (the amplitude A), so we can find the total energy from the maximum potential energy $1/2\,kA^2 = 1/2\,(24.7\,\text{kg/s}^2)(0.085\,\text{m})^2 = 0.0891\,\text{J}$.

We could also find it from the maximum kinetic energy, knowing that the maximum speed is $A\omega$. Then $K_{\max} = 1/2\,mv_{\max}^2 = 1/2\,m(A\omega)^2 = 1/2\,(0.400\,\text{kg})\left((0.085\,\text{m})(7.854\,\text{rad/s})\right)^2$, which gives the same answer. It better, because $k = m\omega^2$.

C. Maximum speed

Above, I used $A\omega$ as the formula for maximum speed, which gives $(0.085\,\text{m})(7.854\,\text{rad/s}) = 0.668\,\text{m/s}$.

We could also back out the speed from the maximum kinetic energy, which is the same as the maximum total energy. Then $v = \sqrt{2K/m} = \sqrt{(0.1783\,\text{J})/(0.400\,\text{kg})} = 0.668\,\text{m/s}$.

5. Transverse waves in a piano wire

The wire is 0.665 meters long, its length density is 6.2 g/m, and its tension is 750 newtons.

A. Propagation speed

The propagation speed of a transverse string wave is $v = \sqrt{F/\mu} = \sqrt{(750\,\mathrm{N})/(6.2\times10^{-3}\,\mathrm{kg/m})} = 347.8\,\mathrm{m/s}$. This is about the same as the speed of sound in air, but that is only a coincidence.

B. Wavelength of fundamental standing wave

The fundamental standing wave has nodes only at the ends, making the distance between adjacent nodes 0.665 meters. The wavelength is twice that: 1.33 meters.

C. Frequency of fundamental standing wave

Here we can use $v = \lambda f$ and solve for f: $f = v/\lambda = (347.8\,\mathrm{m/s})/(1.33\,\mathrm{m}) = 261.5\,\mathrm{Hz}$.

D. Wavelength of first overtone wave

The first overtone has two half-waves in the string length, making its distance between nodes just 0.665/2 meters. We double that to find the wavelength, giving us 0.665 meters.

E. Frequency of first overtone wave

At half the wavelength and the same propagation speed, the frequency of the overtone is twice the frequency of the fundamental, or 523.0 Hz.

6. Tuning a violin

The violin begins at 4 Hz different from the oboe, so it's initially either at 436 Hz or at 444 Hz. Increasing the string tension gives a higher frequency, so because the violin's frequency became more different from the oboe means that it began at a higher frequency, which must have been 444 Hz.

7. Speaking in the lecture hall

A. Intensity at 60 decibels

We're looking for the intensity I in watts per square meter. We have

$$\beta = (10\,\mathrm{dB})\log_{10}(I/I_0),$$

which we solve for I.

$$\begin{aligned}\frac{\beta}{10\,\mathrm{dB}} &= \log_{10}\left(\frac{I}{I_0}\right)\\ 10^{\beta/(10\,\mathrm{dB})} &= I/I_0\\ I &= I_0 10^{\beta/(10\,\mathrm{dB})}\\ &= (10^{-12}\,\mathrm{W/m^2})10^{60/10}\\ &= 10^{-6}\,\mathrm{W/m^2}\end{aligned}$$

B. 80 students speak at once

This gives an intensity (watts per square meter, not decibels) 80 times greater than one student alone.

$$I = 80\cdot10^{-6}\,\mathrm{W/m^2} = 8\times10^{-5}\,\mathrm{W/m^2}$$

C. 80 students at once, in decibels

$$
\begin{aligned}
\beta &= (10\,\mathrm{dB})\log_{10}(I/I_0) \\
&= (10\,\mathrm{dB})\log_{10}\left(\frac{8\times 10^{-5}}{10^{-12}}\right) \\
&= (10\,\mathrm{dB})\log_{10}\left(8\times 10^{7}\right) \\
&= (10\,\mathrm{dB})\left(\log_{10} 8 + 7\right) \\
&= (10\,\mathrm{dB})(0.90 + 7) \\
&= 79.0\,\mathrm{dB}
\end{aligned}
$$

8. Speaking sound power

We are given a decibel level of $\beta = 60$ decibels at a distance of $r = 1$ meter and asked for the sound power P in watts.

Intensity I in watts per square meter depends on distance r and sound power P as

$$I = \frac{P}{4\pi r^2}.$$

Decibels depends on intensity in watts per square meter as

$$\beta = (10\,\mathrm{dB})\log_{10}(I/I_0)\,.$$

To find sound power, we combine these formulas.

$$
\begin{aligned}
I &= I_0 10^{\beta/(10\,\mathrm{dB})} \\
\frac{P}{4\pi r^2} &= I_0 10^{\beta/(10\,\mathrm{dB})} \\
P &= 4\pi r^2 I_0 10^{\beta/(10\,\mathrm{dB})} \\
&= 4\pi(1\,\mathrm{m})^2(10^{-12}\,\mathrm{W/m^2})(10^6) \\
&= 1.26\times 10^{-5}\,\mathrm{W}
\end{aligned}
$$

That's not very much, which tells us that our voices are not very efficient at producing sound.

9. Fire truck from behind

The formula for Doppler frequency shift is

$$f_D = f_S \frac{v - v_D}{v - v_S}$$

where the direction from the source to the detector sets the positive directions for v, v_S, and v_D.

Here we are given $v = 342\,\mathrm{m/s}$, $v_D = 5.0\,\mathrm{m/s}$, $f_S = 120\,\mathrm{Hz}$, and $f_D = 125\,\mathrm{Hz}$. We also know that the source is behind the detector.

A. Direction of source velocity

The detector (Branlee) is moving in the same direction as the sound, so her motion tends to make the detected frequency lower. However, the detected frequency is *higher* than the source frequency, so the fire truck must be traveling toward Branlee, and at a faster speed than Branlee is traveling away.

B. Source speed

We solve the frequency shift for the unknown v_S.

$$\begin{aligned}
f_D &= f_S \frac{v - v_D}{v - v_S} \\
f_D v - f_D v_S &= f_S v - f_S v_D \\
-f_D v_S &= f_S v - f_S v_D - f_D v \\
f_D v_S &= -f_S v + f_S v_D + f_D v \\
v_S &= \frac{-f_S v + f_S v_D + f_D v}{f_D} \\
&= (v_D - v) f_S / f_D + v \\
&= v - (v - v_D) f_S / f_D \\
&= 342\,\mathrm{m/s} - (337\,\mathrm{m/s})(120/125) \\
&= (342 - 323.5)\,\mathrm{m/s} \\
&= 18.5\,\mathrm{m/s}
\end{aligned}$$

10. Simple pendulum

We are given the pendulum's mass m and period T and asked its length L. The period of a pendulum depends on its length and the gravitational field g; the mass does not matter. The simplest formula gives the angular frequency by $\omega^2 = g/L$, the period figures in by $\omega = 2\pi/T$. Thus

$$L = g/\omega^2 = \frac{g}{(2\pi/T)^2} = g\left(\frac{T}{2\pi}\right)^2 = (9.8\,\mathrm{m/s^2})\left(\frac{8\,\mathrm{s}}{2\pi}\right)^2 = (9.8 \cdot 1.273^2)\,\mathrm{m} = 15.9\,\mathrm{m}$$

11. Moon, Earth, and Sun

These calculations are straightforward but tedious. We are given the masses of all three bodies and the distance of the Moon from the Sun and Earth. The formula for the force of gravitational attraction between two bodies is

$$F = G\frac{m_1 m_2}{r^2},$$

so we need to plug in the appropriate masses and distances.

A. Moon and Sun

$$F = \left(6.672 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right) \frac{(7.35 \times 10^{22}\,\mathrm{kg})(1.99 \times 10^{30}\,\mathrm{kg})}{(1.50 \times 10^{11}\,\mathrm{m})^2} = 4.34 \times 10^{20}\,\mathrm{N}$$

B. Moon and Earth

$$F = \left(6.672 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right) \frac{(7.35 \times 10^{22}\,\mathrm{kg})(5.976 \times 10^{24}\,\mathrm{kg})}{(3.82 \times 10^{8}\,\mathrm{m})^2} = 2.01 \times 10^{20}\,\mathrm{N}$$

Even though the Moon is much closer to Earth than it is to the Sun, the Sun attracts it twice as strongly because of its much greater mass.