PHYS 1110 Exam 2
Brief Solutions

Question 1. Pulling stacked blocks

1A. The force accelerating the top block is static friction. The maximum value of this force is $\mu_s m_1 g$. The acceleration of the top block from this force will be $f/m_1 = \mu_s m_1 g/m1 = \mu_s g$.

1B. We figured that in part a: $f = \mu_s m_1 g$.

1C. The pulling force must accelerate *both* blocks at the rate of $\mu_s g$. By Newton's second law, $F = ma = (m_1 + m_2)\mu_s g$.

Question 2. Training centrifuge

2A. One revolution is the circumference of the circle, $2\pi r = 2\pi(4.00\text{ m}) = 25.1$ m.

2B. Period is the time of one cycle. The time of 30 cycles is 60 seconds, so the time of one cycle is 2 seconds.

2C. Angular speed is radians per time. One cycle is 2π radians, and it happens in 2 seconds. So $\omega = 2\pi$ radians/2 s $= \pi$ radians/s.

2D. e. Acceleration is inward (centripetal) for uniform circular motion.

2E. We have a number of formulas that we could use for the magnitude of the acceleration. The easiest one it $a = \omega^2 r$. We were given r, and in a previous question we found ω. So $a = (\pi/\text{s})^2(4.00\text{ m}) = 4\pi^2\text{ m/s}^2 = 39.5\text{ m/s}^2$.

Question 3. Sirius A and B

For gravitational force, we use Newton's gravitational formula.

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.674\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(4.10\times10^{30}\text{ kg})(2.02\times10^{30}\text{ kg})}{(3.00\times10^{12}\text{ m})^2} = 6.14\times10^{25}\text{ N}$$

Question 4. Asteroid and its moon

We are given a radius and a period. It then makes sense to use the centripetal force formula that is expressed in terms of period: $F = ma = m4\pi^2 r/T^2$, where m is the mass of the moon, r is its orbital radius, and T is its orbital period. Equate this to the gravitational attraction between the asteroid and its moon: $F = GmM/r^2$, where M is the mass of the asteroid. Then we have

$$\begin{aligned}
m4\pi^2 r/T^2 &= GmM/r^2 \\
4\pi^2 r^3 &= GMT^2 \\
M &= \frac{4\pi^2 r^3}{GT^2} \\
&= \frac{4\pi^2(50000\text{ m})^3}{(6.674\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(35100\text{ s})^2} \\
&= 6.00\times10^{16}\text{ kg}
\end{aligned}$$

Question 5. Unit of work

The SI unit of work is the joule, which formulates to N·m. But the question asked only for the name.

Question 6. The Nature of work

As a dot product, work must be a scalar.

Question 7. Dot product of vectors

The angle between the vectors is the angle made by placing the vector arrows tail-to-tail. In the diagram, this would construct the angle labeled α. $\vec{A} \cdot \vec{B} = AB \cos \alpha$, which is choice a.

Question 8. Dot products

8A. e. Dot products are scalars. Scalars do not have directions.

8B. a. The formula $\vec{A} \cdot \vec{B} = AB \cos \alpha$ gives its greatest value when $\cos \alpha$ is greatest, which is when $\alpha = 0$. This means the vectors point in the same direction.

8C. a. The dot product operation is commutative, $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.

8D. $(-4, 2) \cdot (2, 5) = -4 \cdot 2 + 2 \cdot 5 = -8 + 10 = 2$.

Question 9. Work pushing a lawnmower

The displacement vector $\vec{s}$ is a horizontal 180 meters, and the force vector $\vec{F}$ is 44.0 N at an angle of 25 degrees below horizontal, so the angle θ between the two vectors is 25°. Thus $\vec{F} \cdot \vec{s} = Fs \cos \theta = (180 \text{ m})(44.0 \text{ N}) \cos 25° = 7178$ J.

Question 10. Flying up Mount Thor

Gravitational potential energy is $U = mgh$; the change is $\Delta U = mgh_2 - mgh_1 = mg(h_2 - h_1) = (0.4 \text{ kg})(9.8 \text{ N/kg})(1200 \text{ m}) = 4704$ J.

Question 11. Flexing a diving board

11A. The force applied to the board is $F = mg = (55 \text{ kg})(9.8 \text{ N/kg}) = 539$ N. The board's displacement is $x = 0.25$ m. Hooke's law tells us $F = -kx$, so $k = -F/x = -(-539 \text{ N})/(0.25 \text{ m}) = 2156$ N/m. The double negative is because the force applied *to* the board is 539 newtons, the force exerted *by* the board is -539 newtons.

Potential energy of the board is $1/2kx^2 = 1/2(2156 \text{ N/m})(0.25 \text{ m})^2 = 67$ J. Alternatively, we could substitute our algebraic formula for k, $k = F/x$, giving $1/2kx^2 = 1/2(F/x)x^2 = 1/2Fx = 1/2(539 \text{ N})(0.25 \text{ m}) = 67$ J.

Question 12. Work sledding down a hill

12A. This question can be answered using the work-energy theorem. We are told the work done by all the individual forces; the net work is just their sum, $(1372 - 819 + 0)$ J = 553 J. The kinetic energy change is equal to the total work done, again 553 J. It is positive, so Annie has a higher kinetic energy at the bottom of the hill than she had at the top. (At the top of the hill, her kinetic energy was zero.)

12B. Her kinetic energy at the bottom of the hill is 553 J. From $K = 1/2mv^2$ we obtain $v = \sqrt{2K/m} = \sqrt{(1106 \text{ J})/(35 \text{ kg})} = \sqrt{31.6 \text{ m}^2/\text{s}^2} = 5.62$ m/s.

Question 13. Impulse sledding down a hill

13A. Here we can use the impulse-momentum theorem $\Delta\vec{p} = \vec{F}t$. The impulse $Ft = (13.8 \text{ N})(3.2 \text{ s}) = 44.2 \text{ kg} \cdot \text{m/s}$, so the momentum change is also $44.2 \text{ kg} \cdot \text{m/s}$. Starting from rest, the final momentum wil be $44.2 \text{ kg} \cdot \text{m/s}$.

13B. From $\vec{p} = m\vec{v}$ we obtain $\vec{v} = \vec{p}/m = (44.2 \text{ kg} \cdot \text{m/s})/(35 \text{ kg}) = 1.26$ m/s.

Question 14. Snowman pinball

This problem, with only conservative forces acting, begs to be analyzed with conservation of mechanical energy. Gravitational potential energy does not change, so the forms of energy here are kinetic energy of the snowman and elastic potential energy of the spring.

$$\begin{aligned}
1/2\ mv_1^2 + 1/2\ kx_1^2 &= 1/2\ mv_2^2 + 1/2\ kx_2^2 \\
1/2\ mv_0^2 + 1/2\ k(0) &= 1/2\ m(0)^2 + 1/2\ kx^2 \\
mv_0^2 &= kx^2 \\
x^2 &= v_0^2\ m/k \\
x &= \pm v_0 \sqrt{m/k} \\
&= \pm(2.10 \text{ m/s})\sqrt{(42 \text{ kg})/(1700 \text{ N/m})} \\
&= \pm 0.33 \text{ m}
\end{aligned}$$

We want to know how far the spring was compressed. Whether that is the positive or negative solution depends on the coordinate system we decide to use. Either way, the spring is *compressed* 0.33 meters. The other solution is how far the spring extends if the snowman stays connected to the end of the spring, oscillating back and forth.

Question 15. Vehicles approaching an intersection

The momentum of Vehicle A is $\vec{p}_A = m_A\vec{v}_A = 1320 \text{ kg}(14.0 \text{ m/s}, 0) = (18480, 0) \text{ kg} \cdot \text{m/s}$. The momentum of vehicle B is $\vec{p}_B = m_B\vec{v}_B = (1700 \text{ kg})(0, 12.0 \text{ m/s}) = (0, 20400) \text{ kg} \cdot \text{m/s}$. The total momentum is thus $\vec{p}_A + \vec{p}_B = (18480, 20400) \text{ kg} \cdot \text{m/s}$.

15A. The x component of the total momentum is 18480 kg · m/s,

15B. The y component of the total momentum is 20400 kg · m/s,