PHYS 1110 Exam 3
Brief Solutions

Question 1. Elastic collision in one dimension

There isn't time on an exam to derive the formula for the outcome of an elastic collision, though you could employ some shortcuts like handling the collision in the center-of-mass frame of reference. But the formulas for the final velocities of both particles in this situation are given in the formula sheet.

A.

$$\begin{aligned} v_{1f} &= v_{1i}\frac{m_1 - m_2}{m_1 + m_2} + v_{2i}\frac{2m_2}{m_1 + m_2} \\ &= (2.00 \text{ m/s})\frac{-250}{750} + (0.50 \text{ m/s})\frac{1000}{750} \\ &= -2/3 \text{ m/s} + 2/3 \text{ m/s} \\ &= 0 \text{ m/s} \end{aligned}$$

B.

$$\begin{aligned} v_{2f} &= v_{2i}\frac{m_2 - m_1}{m_1 + m_2} + v_{1i}\frac{2m_1}{m_1 + m_2} \\ &= (0.50 \text{ m/s})\frac{250}{750} + (2.00 \text{ m/s})\frac{500}{750} \\ &= 1/6 \text{ m/s} + 4/3 \text{ m/s} \\ &= 9/6 \text{ m/s} \\ &= 1.5 \text{ m/s} \end{aligned}$$

The masses and initial velocities are arranged just so that the lighter cart stops dead in the collision.

It's always good, if time permits, to check the solution. In this case that would mean to check that the outcome really *does* conserve momentum and kinetic energy, or at least that the relative speeds are the same before and after the collision.

The easiest of these to check is the relative speeds. Before the collision, the carts approach each other at 1.50 m/s; after the collision, they depart at 1.50 m/s. That's a good sign.

The next easiest to check is total momentum. Before the collision,

$$\sum p = (250 \text{ g})(2.00 \text{ m/s}) + (500 \text{ g})(0.50 \text{ m/s}) = (500 + 250) \text{ g} \cdot \text{m/s} = 750 \text{ g} \cdot \text{m/s}$$

After the collision,

$$\sum p = (500 \text{ g})(1.50 \text{ m/s}) = 750 \text{ g} \cdot \text{m/s}$$

Yes, total momentum is conserved.

Checking total kinetic energy is straightforward but tedious.

$$\begin{aligned} K_i &= \frac{1}{2}(250 \text{ g})(2.00 \text{ m/s})^2 + \frac{1}{2}(500 \text{ g})(0.50 \text{ m/s})^2 = 500 \text{ g} \cdot \text{m}^2/\text{s}^2 + 62.5 \text{ g} \cdot \text{m}^2/\text{s}^2 = 562.5 \text{ g} \cdot \text{m}^2/\text{s}^2 \\ K_f &= 0 + \frac{1}{2}(500 \text{ g})(1.50 \text{ m/s})^2 \ = 562.5 \text{ g} \cdot \text{m}^2/\text{s}^2 \end{aligned}$$

Sure enough, kinetic energy is conserved.

Question 2. Elastic collision statements

All statements are true except for d. If the particles always have the same velocity, there isn't a collision; they're just traveling together.

Question 3. Length of a simple pendulum

The formula on the formula sheet, $\omega^2 = g/L$, relates length L to gravitational field g and angular frequency ω. We want to find L, but in terms of period, not angular frequency. But $T = 2\pi/\omega$, so we obtain

$$L = g/\omega^2 = \frac{g}{(2\pi/T)^2} = \frac{T^2 g}{(2\pi)^2} = \frac{(60\text{ s})^2(9.8\text{ m/s}^2)}{(2\pi)^2} = \frac{3600 \cdot 9.8}{4\pi^2}\text{ m} = 894\text{ m}$$

Question 4. Totally inelastic collision statements

These are the same statements we evaluated above in Question 2 for elastic collisions. But for a totally inelastic collision, kinetic energy is not conserved, just momentum. So statement a is not true. Statement b is still true. Statement c is no longer true: in a totally inelastic collision, relative speeds are zero ofter the collision, but not before. Statement d is still not true: same velocity before and after still means no collision. Statement e is still true: velocity of center of mass is total momentum divided by total mass, and both total momentum and total mass are the same after the collision as before.

Question 5. Grandpa's grinding stone

A. Moment of inertia

The grinding stone is a uniform cylinder.

$$I = 1/2MR^2 = 1/2(25\text{ kg})(0.20\text{ m})^2 = 0.50\text{ kg}\cdot\text{m}^2$$

B. Angular speed

This is a unit conversion problem. We need to convert revolutions to radians and minutes to seconds.

$$50\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi\text{ rad}}{\text{rev}} \cdot \frac{\text{min}}{60\text{ s}} = 5.24\frac{\text{rad}}{\text{s}}$$

C. Rotational kinetic energy

$$K = 1/2\ I\omega^2 = 1/2\ (0.50\text{ kg}\cdot\text{m}^2)\left(5.24\frac{\text{rad}}{\text{s}}\right)^2 = 6.85\text{ J}$$

D. Angular acceleration

Rate of change of angular velocity

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{5.24\text{ rad/s}}{12\text{ s}} = 0.436\text{ rad/s}^2$$

E. Torque

Torque $\tau = I\vec{\alpha} = (0.50\text{ kg}\cdot\text{m}^2)(0.436\text{ rad/s}^2) = 0.218\text{ N}\cdot m$.

F. Work

There are a few ways to calculate the work done. First, I'll use $\tau\Delta\theta$. We know the torque is $\vec{\tau} = \vec{r}\times\vec{F} = (0.20\text{ m})(15\text{ N}) = 3.0\text{ N}\cdot\text{m}$. Then the angle traversed is $\Delta\theta = \omega\Delta t = (5.236\text{rad/s})(40.0\text{ s}) = 209.44$ radians, so $W = \tau\Delta\theta = (3.0\text{ N}\cdot\text{m})(209.44) = 628$ J.

Another way to answer this avoids rotational quantities and goes back to our original formula for work: force applied along a distance. Here the force is the 15 newtons of friction, and the distance is the distance the tool drags along the rim of the grinding stone. The grinding stone has a rotational speed of 50 rev/min, and one revolution is a distance of $2\pi r = 2\pi(0.20 \text{ m}) = 1.257$ m. That comes out to 62.83 meters per minute or 1.047 meters per second. In 40.0 seconds, the tool drags across 41.89 meters, so the work done is (15 N)(41.89 m) = 628 J, as we also found above.

Question 6. Drop a ring on a rotor

A. Angular momentum

Angular momentum $L = I\omega = (0.0144 \text{ kg} \cdot \text{m}^2)(12.0 \text{ rad/s}) = 0.1728 \text{ kg} \cdot \text{m/s}$.

B. Statements about the rotor

This is a totally inelastic angular collision. Dropping the ring on the rotor does not produce a torque on the system, but the ring and rotor apply equal and opposite torques on each other. Thus, angular momentum is conserved for the system of (ring + rotor). Therefore, the only true statement is c.

C. Final angular speed

We use conservation of angular momentum.

$$I_1\omega_1 = (I_1 + I_2)\,\omega_2$$
$$\omega_2 = \omega_1 \frac{I_1}{I_1 + I_2} = (12.0 \text{ rad/s})\frac{0.0144}{0.0144 + 0.0096} = 12.0\frac{0.0144}{0.024} \text{ rad/s} = 7.2 \text{ rad/s}$$

Question 7. Mass oscillating on a spring

We are given mass $m = 0.250$ kg, period $T = 1.50$ s, and maximum speed $v_{\text{max}} = 0.5$ m/s. We know we'll want to convert period to angular frequency, so $\omega = 2\pi/T = 4.189$ rad/s. We also know that maximum speed has the formula $v_{\text{max}} = A\omega$, so amplitude must be $A = v_{\text{max}}/\omega$.

A. Amplitude

The first quantity we are asked to find is amplitude A. From the above, we have $A = v_{\text{max}}/\omega = (0.5 \text{ m/s})/(4.189 \text{ rad/s}) = 0.119$ m.

B. Spring constant

We know for an oscillating mass $\omega^2 = k/m$, so $k = m\omega^2 = (0.250 \text{ kg})(4.189 \text{ rad/s})^2 = 4.387 \text{ kg/s}^2$. In case you are wondering about the units, reaiize that if you multiply kg/s^2 by m/m, you get N/m.

C. Where kinetic energy is maximum

That is at the middle of the oscillation.

D. Maximum kinetic energy

We are given the maximum speed, so use $K = 1/2\ mv^2 = 1/2\ (0.250 \text{ kg})(0.5 \text{m/s})^2 = 0.03125$ J.

E. Where potential energy is maximum

Spring potential energy is $1/2\ kx^2$. This is greatest at the farthest distance from equilibrium.

F. Maximum potential energy

There are several ways to find this. The most elegant is to use conservation of mechanical energy; where kinetic energy is zero, the potential energy must be equal to the total mechanical energy, which must be equal to the kinetic energy when the spring is at its equilibrium position. This is the maximum kinetic energy of 0.03125 J.

A more direct way to find the maximum potential energy is to simply find the maximum value of $1/2\ kx^2$. The value of k does not change, so we need to maximize x^2. The most extreme value of x is $\pm A$, so $U_{\text{max}} = 1/2\ kA^2 = 1/2\ (4.387\ \text{kg/s}^2)(0.119\ \text{m})^2 = 0.031$ J. There is some roundoff error here. Recall that we found k and A from v_{max} and T; if we substitute the formulas we used, we obtain $1/2\ kA^2 = 1/2\ m\omega^2(v_{\text{max}}/\omega)^2 = 1/2\ mv_{\text{max}}^2$, identically the maximum kinetic energy.

G. Where mechanical energy is maximum

No non-conservative forces are acting, so mechanical energy is not changing. Thus total mechanical energy is the same everywhere.

Question 8. Kepler's equal area law

The equal area law is a consequence of conservation of angular momentum.

Question 9. Bosendorfer C_0

We know at the outset that the wave speed is determined by tension F and length density μ. We are given the wave frequency $f = 16.3516$ Hz and tension $F = 600$ N.

A. Wave speed

We are given the further clue that wavelength $\lambda = 4.20$ m, so we use $u = \lambda f = (4.20\ \text{m})(16.3516\ \text{Hz}) = 68.68$ m/s.

B. Mass

We find mass m from the length density $\mu = m/L$. Unfortunately, we only know L, so we need to find μ. This comes from $u = \sqrt{F/\mu}$. We find $\mu = F/u^2$, and

$$\begin{aligned} m &= \mu L \\ &= FL/u^2 \\ &= \frac{(600\ \text{N})(2.10\ \text{m})}{\left[2 \cdot (2.10\ \text{m})(16.3516\ \text{Hz})\right]^2} \\ &= \frac{(600\ \text{N})}{(2.10\ \text{m})\left[2 \cdot (16.3516\ /\text{s}^2)\right]^2} \\ &= 0.267\ \text{kg} \end{aligned}$$

C. Wave type

Striking the string transversely produces a transverse wave in the string.

Question 10. Beats

The beat frequency is equal to the difference in frequencies of the combining notes. Tightening the violin string makes the beat frequency higher, so that means that tightening the violin string makes the violin's frequency more different from the oboe's frequency. To match the oboe's frequency, then, the violin's string should become looser.

Question 11. Firecracker

A. Sound intensity at the student's ear

The formula for sound intensity in watts per meter at distance r from a source with power P watts is $I = P/(4\pi^2)$. We don't know the power P; instead, we are tasked with using the intensity I_1 at distance R_1 to calculate the intensity I_2 at another distance r_2. Our approach will be to find the power in terms of I_1 and r_1 and apply it to r_2.

$$I_1 = \frac{P}{4\pi r_1^2}; \quad I_2 = \frac{P}{4\pi r_2^2}$$
$$4\pi r_1^2 I_1 = P = 4\pi r_2^2 I_2$$
$$I_2 = I_1 \; r_1^2/r_2^2$$

B. Decibel level at instructor's ear

Decibel level differences are logarithmically related to intensity level ratios. We start with the formula

$$\beta_2 - \beta_1 = (20 \text{ dB}) \log_{10}(r_1/r_2),$$

which applies to sound from the same source detected at two different distances r_1 and r_2. In this case we aren't trying to find $\beta_2 - \beta_1$; we want β_1 alone. So

$$\begin{aligned} \beta_1 &= \beta_2 - (20 \text{ dB}) \log_{10}(r_1/r_2) \\ &= 110 \text{ dB} - (20 \text{ dB}) log_{10}(0.30/1.80) \\ &= 110 \text{ dB} - (-15.56 \text{ dB}) \\ \beta_1 &= 126 \text{ dB} \end{aligned}$$

Question 12. Train whistle

A. Effect of Train A's speed

The approaching train's speed shortens the wavelength, which raises the frequency.

B. Effect of Train B's speed

The train traveling into the wavefronts effectively increases the sound speed, raising the frequency.

C. Detected frequency

The Doppler formula is

$$f_D = f_S \frac{v - v_D}{v - v_S}$$

The formula as written isn't exactly what we need, though. We know $f_D = 651.3$ Hz, $f_S = 550$ Hz, $v = 342$ m/s, and $v_S = 38.0$ m/s; we want to find v_D. Some algebra gives us

$$v_D = v - \frac{f_D}{f_S}(v - v_S) = 342 \text{ m/s} - \frac{651.3}{550}(304 \text{ m/s}) = (342 - 360) \text{ m/s} = -18 \text{ m/s}.$$

The negative sign indicates that the detector (Train B) is traveling opposite the direction that the sound wave fronts are moving. In other words, Train A and Train B are moving toward each other. Train B's speed is 18 meters per second.