PHYS 1110 Quiz 1 (of 1)

Brief Solutions

1. Average velocity

The units are m/s.

2. Momentum

The units are kg · m/s.

3. Convert parsecs to light-years

You are given the equivalents of parsecs in meters and light-years in meters. Formally, you will need to convert parsecs to meters and then meters to light-years. You can do this with two factors in the formula.

$$11.25 \text{ pc} \cdot \frac{3.094 \times 10^{16} \text{ m}}{\text{pc}} \cdot \frac{\text{light-year}}{9.460 \times 10^{15} \text{ m}} = 36.79 \text{ light-years}$$

4. Vector $(-7.00 \text{ m/s}, 1.00 \text{ m/s})$

a. Magnitude

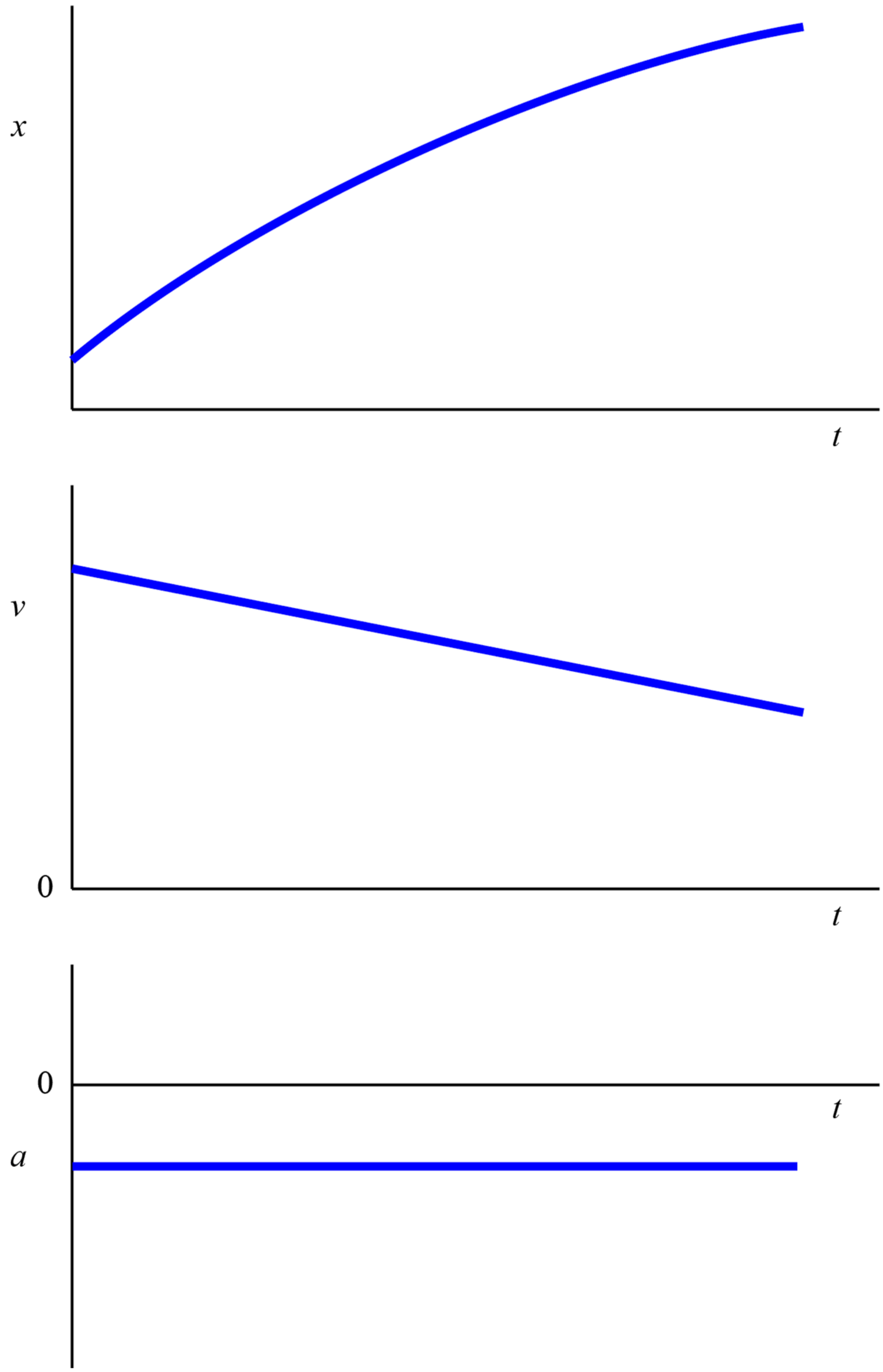
$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-7 \text{ m/s})^2 + (1 \text{ m/s})^2} = \sqrt{50} \text{ m/s}$$

b. Angle

$\theta = \arctan(A_y/A_x) = -8.13°$. But this isn't correct: the vector points in the $-x$ direction, while this angle is in the $+x$ direction, When the x component of a vector is negative, you need to add 180° to the result of the inverse tangent function. Thus $\theta = 180° + \arctan(A_y/A_x) = 180° - 8.13° = 171.9°$.

5. Cyclist coasting uphill

a, b, and c. The acceleration should be constant and negative (or at least the opposite sign of the velocity). The velocity should be positive throughout and steadily decreasing. The position-time graph should be increasing throughout (because Ada is moving forward), with a decreasing slope; concave downward.



6. Cyclist coasting uphill, with numbers

The numbers are $v_0 = 12.0$ m/s, $x - x_0 = 120$ m, and $a = -0.30$ m/s^2.

a. Final speed

We aren't given time, so we can use the formula $v^2 - v_0^2 = 2a(x - x_0)$, solved for v.

$$\begin{aligned} v^2 &= v_0^2 + 2a(x - x_0) \\ &= (12 \text{ m/s})^2 - (0.6\text{m/s}^2)(120 \text{ m}) \\ &= 144 \text{ m}^2/\text{s}^2 - 72 \text{ m}^2/\text{s}^2 \\ v^2 &= 72 \text{ m}^2/\text{s}^2 \\ v &= 6\sqrt{2} \text{ m/s} = 8.49 \text{ m/s} \end{aligned}$$

b. Travel time

The simplest way to approach this question, since we now know the initial velocity, final velocity, and acceleration, is to solve the definition of average acceleration for time.

$$a = \frac{\Delta v}{\Delta t}$$
$$\Delta t = \frac{\Delta v}{a} = \frac{8.49 \text{ m/s} - 12.0 \text{ m/s}}{-0.30 \text{ m/s}^2} = 11.7 \text{ s}$$

Since we also know the distance traveled and the initial and final velocities, we can also use a different kinematic equation if we choose.

$$x - x_0 = 1/2\ (v_0 + v)t$$
$$t = \frac{2(x - x_0)}{v_0 + v} = \frac{240 \text{ m}}{12 \text{ m/s} + 8.49 \text{ m/s}} = 11.7 \text{ s}$$

Alternatively, if we didn't know the final velocity, we could solve $x - x_0 = v_0 t + 1/2\ at^2$ for t. For this, we need the quadratic formula.

$$0 = t^2 + 2\frac{v_0}{a}t - \frac{2(x - x_0)}{a}$$
$$t = -\frac{v_0}{a} \pm \sqrt{\left(\frac{v_0}{a}\right)^2 + \frac{2(x - x_0)}{a}}$$
$$t = -\frac{12 \text{ m/s}}{-0.30\text{m/s}^2} \pm \sqrt{\left(\frac{12 \text{ m/s}}{-0.30 \text{ m/s}^2}\right)^2 + \frac{240 \text{ m}}{-0.30 \text{ m/s}^2}}$$
$$t = 40 \text{ s} \pm \sqrt{1600 \text{ s}^2 - 800 \text{ s}^2}$$
$$t = 40 \text{ s} \pm 28.28 \text{ s} = 68.28 \text{ s} \quad \text{or} \quad 11.72 \text{ s}$$

We want the shorter time. The longer time would model if Ada continued coasting up the hill until she momentarily came to a stop and then coasted back down to the position 120 meters ahead of where she started.

7. Walking around the lake

a. Path length

Add up the segments of the path: 100 m + 300 m + 300 m + 300 m + 50 m = 1050 meters.

b. Distance

In a straight line, from the villager's house to the friend's house: 100 m to the first corner, then 250 m across the lake to the house = 350 meters.

c. Average speed

This is path length divided by time: 1050 m/525 s = 2.0 m/s.

d. Average velocity

This is displacement divided by time: 350 m/525 s = 2/3 m/s.

Second question 7. Velocity of a rolling ball

The speed of the ball is $v = 2.25$ m/s in the direction $\theta = -40°$.

$v_x = v \cos\theta = 1.72$ m/s.

$v_y = v \sin\theta = -1.45$ m/s.

8. Vector multiplied by a scalar

$$7.00 \cdot (3.50 \text{ m}, -2.40 \text{ m}) = \big(7.00 \cdot 3.50 \text{ m}, 7.00 \cdot (-2.40 \text{ m})\big) = (24.5 \text{ m}, -16.8 \text{ m})$$

9. Two displacements

We add together the two displacement vectors.

$$(25 \text{ m}, 40 \text{ m}) + (-30 \text{ m}, 10 \text{ m}) = ((25 - 30) \text{ m}, (40 + 10) \text{ m}) = (-5 \text{ m}, 50 \text{ m})$$

10. Right triangle relationships

In this triangle, side A is the hypotenuse and B and C are the catheti. The angle θ is opposite side C and adjacent to side B.

a. $\sin\theta = C/A$.

b. $\cos\theta = B/A$.

c. $\tan\theta = C/B$.

d. Pythagorean relation $A^2 = B^2 + C^2$.