PHYS 1110 Round 1 Retests

Brief Solutions

Standard 1 Retest 1

Question 1. SI unit for radians

Radians are unitless, as a ratio of distances. So the SI unit is $\text{kg}^0\ \text{m}^0\ \text{s}^0$.

Question 2. SI unit for acceleration

Acceleration is the rate of change of velocity, while velocity is the rate of change of position. The position unit is meters, so its rate of change is meters per second; the rate of change of that would be meters per second per second. So the SI unit is $\text{kg}^0\ \text{m}^1\ \text{s}^{-2}$.

Question 3. SI unit for impulse

Impulse is force applied for a time period, so its units are force units multiplied by time units, newtons times seconds. So the SI unit is $\text{kg}^1\ \text{m}^1\ \text{s}^{-1}$.

Standard 2 Retest 1

Question 1. Barleycorns in a pied

We are given that 1 barleycorn = 1/3 inch and that 1 pied = 326.6 mm. We start with 1 pied and convert to barleycorns.

$$1\ \text{pied} \cdot \frac{326.6\ \text{mm}}{1\ \text{pied}} \cdot \frac{1\ \text{in}}{25.4\ \text{mm}} \cdot \frac{3\ \text{barleycorn}}{1\ \text{in}} = 38.57\ \text{barleycorn}$$

Question 2. One furlong per minute in meters per second

We are given that 8 furlongs = 1 mile; 1 mile = 5280 ft; 1 meter = 3.28 ft; and 60 s = 1 min. We start with One furlong per minute and convert to meters per second.

$$\frac{1\ \text{furlong}}{1\ \text{min}} \cdot \frac{1\ \text{mi}}{8\ \text{furlong}} \cdot \frac{5280\ \text{ft}}{\text{mi}} \cdot \frac{1\ \text{m}}{3.28\ \text{ft}} \cdot \frac{1\ \text{min}}{60\ \text{s}} = 3.35\ \text{m/s}$$

Question 3. One furlong in yards

We are given that 1 mile = 5280 feet, 8 furlongs = 1 mile, and 1 yard = 3 feet. We start with one furlong and convert to yards.

$$1\ \text{furlong} \cdot \frac{1\ \text{mi}}{8\ \text{furlong}} \cdot \frac{5280\ \text{ft}}{1\ \text{mi}} \cdot \frac{1\ \text{yd}}{3\ \text{ft}} = 220\ \text{yd}$$

Standard 3 Retest 1

Question 1. Position-time plots and descriptions

You were shown four position-time plots and directed to match them to verbal descriptions of the same motion. The precise plots were randomly selected, so it's not possible to give a universally relevant explanation here. Remember that a position-time plot shows where something is as time progresses.

Question 1. Velocity-time plots and position-time plots

Here you needed to select the position-time plot corresponding to the given velocity-time plot. Here's some things to keep in mind as you choose.

- When a velocity-time plot is zero, the position-time plot should be flat (slope of zero).
- When a velocity-time plot is flat (constant), the position-time plot should be a straight line (constant slope).
- When a velocity-time plot is changing (not flat), the position-time plot will be curved (changing slope).
- When a velocity-time plot is positive (above zero), the position should be increasing (more positive or less negative).
- When a velocity-time plot is negative (below zero), the position should be decreasing (less positive or more negative).

Standard 4 Retest 1

Question 1. Average speed to work

Average speed is path length (distance by road) divided by time. You'll need to convert kilometers to meters.

Question 2. Average velocity round-trip to work

Average velocity is displacement divided by time. He finished where he started (at home), so his displacement is zero.

Question 3. Meaning of negative velocity

Velocity is the rate of change of position. The position must be decreasing (becoming less positive or more negative).

Standard 5 Retest 1

Question 1. The hare catching the tortoise

The tortoise, moving at speed v_t, is a distance x_t from the finish. The hare is a distance x_h behind the tortoise. The tortoise will reach the finish in time $t = x_t/v_t$. To tie the tortoise, the hare must travel a distance $x_h + x_t$ in time t. We will need to travel at speed $(x_h + x_t)/t = v_t(x_h + x_t)/x_t$.

Question 2. Ball at constant acceleration

The equation of motion of the ball is $x = 1/2\ at^2$. We are given the time t_1 for some distance x_1 and asked for the time t_2 for a different distance x_2. We know that the accelerations a are the same. So

$$\begin{aligned} a &= 2x/t^2 \\ 2x_1/t_1^2 &= 2x_2/t_2^2 \\ x_1 t_2^2 &= x_2 t_1^2 \\ t_2^2 &= t_1^2 x_2/x_1 \\ t_2 &= \pm\ t_1\sqrt{x_2/x_1} \end{aligned}$$

We don't need to consider the negative solution, because the equation of motion did not apply before time zero, so $t_2 = t_1\sqrt{x_2/x_1}$.

Standard 6 Retest 1

Question 1. Side with a length formula

The diagram showed a right triangle with one angle θ and sides A and B with hypotenuse C, where side B is opposite angle θ. It gave a formula for the length of a side and asked which side had that length. (The specific question was chosen randomly.) We know that $\sin\theta = B/C$, $\cos\theta = A/C$, and $\tan\theta = B/A$. Solve the formula containing the relevant trigonometric function to find the proper side.

Question 2. Side with a length formula

This question showed the four trigonometric quadrants and asked for the sign of a particular trigonometric function in a particular quadrant. Sine is positive in quadrants 1 and 2 and negative in quadrants 3 and 4; cosine is positive in quadrants 1 and 4 and negative in quadrants 2 and 3; and tangent is positive in quadrants 1 and 3 and negative in quadrants 2 and 4.

Standard 7 Retest 1

Questions 1 and 2. x- and y-components in inclined coordinates

The vector $\vec{w}$ points away from both the $+x$ and $+y$ axes, so both its x and y components will be negative. Constructing a right triangle with hypotenuse $\vec{w}$ and sides parallel to $+x$ and $+y$ finds the components in the inclined coordinates. The x side has length $w \sin\theta$ and the y side has length $w \cos\theta$.

Question 3. Graphically adding vectors

Start with vector $\vec{A}$, then place vector $\vec{B}$'s tail at the head of $\vec{A}$. The sum $\vec{A}+\vec{B}$ starts at the tail of $\vec{A}$ and ends at the head of $\vec{B}$.

Question 4. Components of a vector in polar coordinates

When the polar angle θ is defined according to the trigonometric convention as being counterclockwise of the $+x$ axis, $x = r\cos\theta$ and $y = r\sin\theta$.

Standard 8 Retest 1

Question 1. Quantities that are constant

For projectile motion, $a_x = 0$, $a_y = -g$, and $v_x = v_{0x}$, none of which change with time. The remaining listed quantities are $v_y = v_{0y} - gt$, $x = x_0 + v_{0x}t$, and $y = y_0 + v_{0y}t - 1/2\ gt^2$, all of which do change with time.

Question 2. Quantities that depend on g

Here, we are asked about $v_y = v_{0y} - gt$, $y = y_0 + v_{0y} - 1/g\ gt^2$, $v_x = x_0 + v_{0x}t$, and $x = x_0 + v_{0x}t$. The ones depending on g are y and v_y. For a given initial velocity, x and v_x would progress the same in any gravitational field.

Question 3. Components of initial velocity

Here you were given an angle θ *below* horizontal and asked to find the initial horizontal and vertical components of velocity. We obtain $v_{0x} = v_0 \cos\theta$ and $v_{0y} = -v_0 \sin\theta$.

Standard 9 Retest 1

Question 1. Bomb travel distance

The question asks for the horizontal distance traveled. This will be $x_L = v_{0x}t_L$, where t_L is the time at which the bomb lands on the ground. You need to use the height information to find t_L. The height equation is $y = y_0 + v_{0y}t - 1/2\ gt^2$. To find the time t_L, you need to solve this for t when $y = 0$. It is a quadratic equation, which can be put in the form $o = 1/2\ gt^2 - v_{0y}t_L - y_0$. The quadratic formula yields

$$t_L = \frac{v_{0y} \pm \sqrt{v_{0y}^2 + 2gy_0}}{g}.$$

We see that only the solution with the $+$ of the $\pm$ is meaningful, because the second term in the numerator (the radical) has a larger absolute value than the first term, so only the addition gives a positive landing time. To get started, you need to find the components of initial velocity, $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$. Here you must be careful, because $\theta < 0$: the bomb's initial velocity is partly downward. This means that $v_{0y} < 0$.

Question 2. Bomb travel distance at higher speed

As described in the previous problem, $v_{0y} < 0$. A look at the formula for t_L shows that a greater speed v_0 then makes the bomb hit the ground sooner. But the greater speed also makes the bomb travel horizontally farther each second. Which factor wins? Your intuition probably tells you that a faster speed makes the bomb's arc closer to a straight line, thus traveling farther. But can we find that in the formula? Expanding v_{0x} and v_{0y} in the formula for x_L obtains

$$\begin{aligned} x_L &= v_0 \cos\theta / g(v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gh}) \\ &= \frac{v_0}{g} \cos\theta(v_0 \sin\theta + v_0\sqrt{sin^2\theta + 2gh/v_0^2}) \\ &= \frac{v_0^2}{g} \cos\theta(\sin\theta + \sqrt{sin^2\theta + 2gh/v_0^2}) \end{aligned}$$

Even though $\sin\theta$ inside the parentheses is negative, we see that the entire quantity inside the parentheses is positive. But as v_0 increases, the quantity gets smaller. But as long as it does not decrease as fast as $1/v_0^2$, the quantity multiplied by v_0^2 gets larger. So sure enough, the bomb travels farther before landing when it has a faster initial speed, even though it starts out moving downward.

Question 3. Initial v_x

We are given an initial height above the ground, a horizontal travel distance to landing, and a landing time. The question asks for the horizontal component of the initial velocity. We know that the horizontal component of velocity never changes in a ballistic trajectory, and we know that horizontal distance to landing is given by $x_L = v_x t_L$. Solving for v_x gives $v_x = x_L/t_L$.

Standard 10 Retest 1

Question 1. What is force?

Force is a vector.

Question 2. Matching terms to definitions

Sadly, the answer key had one pair inexplicably swapped. The definitions ought to have been

- Force: An influence on an object that would cause the object's velocity to change
- Net force: The vector sum of all forces acting on an object
- Net force: A force on a body from its interaction with another body
- Inertia: The tendency of an object to resist acceleration by a force

Standard 11 Retest 1

Question 1. Speed after transiting a nebula

The net force on the spaceship while it passed through the nebula was zero. Newton's first law tells us that its velocity would not change during that time, so its final velocity would just be the same as its initial velocity.

Standard 12 Retest 1

Question 1. Net force and acceleration

Net force and acceleration are vectors that are directly proportional to each other. The constant of proportionality between them is the mass, which is always positive. Mathematically, this requires the net force and acceleration vectors to be in the same direction.

Question 1. Net force, acceleration, and mass

We are given the net force and acceleration, and asked to find the rocket sled's *inertial mass*. From $\vec{a} = \vec{F}/m$ we obtain $m = F/a$.

Standard 13 Retest 1

Question 1. Objects interacting with a hockey puck

Three forces were identified: weight, kinetic friction, and a normal force. Every force on any object is a result of an interaction with another object, and is accompanied by an equal and opposite force on the other object.

- The puck's *weight* comes from the puck's gravitational interaction with the Earth. Its partner is an upward force on the Earth.
- The *normal* force results from the ice pushing up on the puck. In turn, the puck pushes down on the ice.
- The force of *friction* is a backwards force on the puck from the ice surface. In turn, the puck pushes the surface of the ice forward.

Standard 14 Retest 1

Question 1. Cyclist

The forces acting on the cyclist are friction from the road pushing him forward (up the hill); wind pushing him downhill (opposite direction of his path); gravity pushing him straight down; and a normal force from the road, perpendicular outward from the road. The net force on the cyclist is zero, so the forces all sum to zero.

Question 2. Rotor

The forces acting on the rider are gravity pushing him straight down; friction pushing straight up; and a normal force pushing horizontally inward. The friction force and weight are equal and opposite, as the rider does not accelerate vertically. The normal force is *not* opposed by any other horizontal forces; the rider *is* accelerating horizontally, toward the rotation axis.

Standard 15 Retest 1

Question 1. Creepy doll part 1

The force of friction is opposite the direction of the doll's velocity.

Question 2. Creepy doll part 2

The floor is level, so the normal force must oppose the doll's entire weight mg. The magnitude of the kinetic friction thus is $\mu_k mg$.

Question 3. Static friction on a slope

The direction of the friction force is parallel to the slope, uphill to cancel the downhill component of the box's weight.

Question 4. Pushing a stationary crate

Static friction is a force of constraint. in this case, it prevents Bruce from accelerating the crate horizontally. No other horizontal forces operate, so the magnitude of the friction force is equal to the magnitude of Bruce's horizontal push. The coefficient of statice friction and the crate's weight set an upper limit to the magnitude of the force of static friction. The actual magnitude of force should have been less than this upper limit.

Standard 16 Retest 1

Question 1. Tension in a wire

The weight of the sphere is mg. This weight ($-y$ direction), a horizontal tension in the $-x$ direction, and an oblique tension force in the direction θ add to a net force of zero. Thus, the vertical component of the oblique tension force $F_{3y} = mg$, and its horizontal component is whatever is dictated by its angle. Trigonometry requires that $\tan\theta = F_{3y}/F_{3x}$, so $F_{3x} = mg/\tan\theta$. Then you can find the magnitude F_3 from $F_3^2 = F_{3x}^2 + F_{3y}^2$. To keep the system static, the horizontal tension F_2 must be opposite F_{3x}, though the question did not ask for F_2.

Question 2. Normal force of a static block on a slope

The normal force opposes the perpendicular component of the block's weight, which is $mg\cos\theta$.

Question 3. Gravitational field strength

The question provides the values of the mass m and the weight w. Knowing that $w = mg$ allows us to find g by solving for it in the formula: $g = w/m$.

Standard 17 Retest 1

Question 1. Buoyancy

We haven't learned anything about the force of buoyancy. But we do know that it must cancel the barge's weight.

Question 2. Push by the deck mate

The deck mate pushes off the wall to cancel the lateral component of the mule's pull. The diagram shows that the tow line makes a 3:4:5 right triangle. The force of tension is in the same direction as the tow line, so it also makes a 3:4:5 right triangle, with a hypotenuse of 2000 newtons. This means that the lateral component has a ratio of 3:5 with the tension, it must be 1200 newtons. Thus the average push by the deck mate is 1200 N. (That is a lot. In practice a longer tow line would be a good idea.

We can't have exhausted all the forces acting on the barge, because the barge isn't accelerating forward. So there must be another force acting backwards on the barge to counteract the forward component of the mule's pull. This force is drag against the water.

Question 3. Banking airplane

The plane's altitude is not changing, and its speed is not changing, but it *is* accelerating: it is in a turn. We know this because its lift pulls it to the side (its left in the diagram) as well as upward. Here, the net force on the plane must be the horizontal component of its lift: everything else cancels.

The vertical component of the plane's lift must be equal to the plane's weight mg. If we call the lift $\vec{L}$, we know that $L_y = L\cos\theta = mg$ and $L_x = L\sin\theta$. Trigonometry tells us that $\tan\theta = L_x/L_y$, so $L_x = L_y\tan\theta = mg\tan\theta$. This is the net force on the plane.

Standard 18 Retest 1

Question 1. Acceleration of a box sliding downhill

We are given the mass m of the box, the slope θ of the hill, and the coefficient of friction μ_k between the box and the hill. We are asked to find the acceleration of the box.

We have seen enough problems of this type that I hope I can safely skip over the critical steps of setting up the inclined coordinate system, finding the components of the box's weight in that system, and finding the formulas for the nornal force and friction. We end up with the forces acting parallel to the incline being the weight, at $+mg\sin\theta$, and friction, at $-\mu_k mg\cos\theta$. (This is setting the $+x$ direction as downhill parallel to the ramp.) Then the box's acceleration is the sum of these forces divided by the mass of the box: $a = g(\sin\theta - \mu_k\cos\theta)$.