Discussion: Straight-line constant acceleration kinematics

Summary

Average acceleration is the rate of the change of an object's velocity.

$$a_{\text{avg}} = \Delta v/\Delta t$$

On a plot of velocity vs. time, a_{avg} is the slope of the line connecting the starting and ending events.

Instantaneous acceleration is the limit of average acceleration as the time interval becomes infinitesimally brief. It is the acceleration at a particular instant of time.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

On a of velocity-time plot, a is the slope of the tangent line at that particular instant. Conversely, the area under an acceleration-time plot is the change in velocity.

Kinematic Formulas

When velocity is constant, $\Delta x/\Delta t$ is the same for any time interval. Then $x = x_0 + vt$.

When acceleration is constant, $v = v_0 + at$; $x = x_0 + v_0 t + \frac{1}{2}at^2$.

Algebraic substitution allows us to find relations not requiring t or not requiring a:

$$2a(x - x_0) = v^2 - v_0^2; \qquad x - x_0 = \frac{1}{2}(v_0 + v)t$$

Problems

1. When you write up a physics problem for a grade, it's important for the grader to understand what you did. Your write-up should show what physics principles you decided are important to understanding the problem; it should show the quantities that you decided are pertinent and the labels you assigned to them; it should show the equations and formulas linking the quantities you know to the quantities you are trying to find; it should show the algebra solving the equations for the desired quantity; and finally, it should show the answer obtained, with units.
   A good written solution comprises:
   - A diagram
   - The physical principle(s) applying to the situation,
   - Explicitly matching the quantities in the problem to the quantities in the formulas (correctly setting up the problem),
   - Solving the solution symbolically, and
   - Showing the correct answer.

   If you present your solution this way, you will convince the grader that you truly understand the physics of the situation. Or, if you make a mistake somewhere, the grader can easily find it, to give you appropriate feedback to help you going forward. Perhaps most importantly,

you will be more likely to catch your mistakes yourself, and you will be less likely to carelessly use a formula that doesn't apply or to substitute the quantities into the wrong place. Admittedly, this process takes time. But would you prefer to find a wrong answer quickly?

Practice it! Use the process to work on the following problem. Start with a diagram showing the subject(s) of the problem, labeled with symbols for the pertinent quantities. List the values of the given quantities. Develop the model with the appropriate equations. Once you obtain the symbolic solution, substitute the given values into your derived formula, and calculate the answer.

Then work the remaining problems using the same process.

2. A ball starts from rest and rolls down an incline at a constant acceleration. In 5.0 s, it rolls a distance of 50.0 m down the hill.
   a. What is its acceleration?

   b. If the same ball rolls down the same incline with the same acceleration, but begins with an initial speed of 2.0 m/s downhill, how far down the hill will it be in 5.0 s?

   c. If the ball begins with an initial speed of 2.0 m/s *uphill*, where will it be in 5.0 s?

3. An automobile accelerates constantly from rest, traveling 400 m in 20.0 s.
   a. What is its average velocity over this interval?

   b. What is its final velocity?

c. The car's engine is adjusted, and the car is again accelerated constantly from rest through a distance of 400 m. This time, its final velocity is 50 m/s. What is the time elapsed?

4. The human body can survive an acceleration trauma incident (sudden stop) if the magnitude of the acceleration is less than 250 m/s^2. If you are in an automobile accident with an initial speed of 30 m/s and you are stopped by an airbag that inflates from the dashboard, over what distance must you stop for you to survive the crash?

5. Serena lobs a tennis ball straight upwards. It rises a distance H before stopping and falling back down. Disregard air resistance.

   a. What is the initial speed v_0 of the tennis ball? Express in terms of height H and g, the acceleration due to gravity.

   b. How much time does it take for the ball to reach the top of its trajectory? Express in terms of H and g.

   c. When the first ball is exactly at the top of its trajectory, Serena lobs a second tennis ball straight up with the same initial speed v_0.

      i. How much time later are the balls at the same height? Express in terms of H and g.

ii. What is the height at which the balls cross? Express in terms of H and g.