Discussion Worksheet 2: Trajectory Kinematics

Objectives

- Represent position, velocity and acceleration as vectors.
- Work with ballistic trajectories in the vertical and horizontal directions.

Summary

Position, velocity, and acceleration vectors

Position $\vec{r} = (x, y, z)$

Velocity $\vec{v} = \lim_{\Delta t \to 0} \left({}^{\Delta x}/_{\Delta t}, {}^{\Delta y}/_{\Delta t}, {}^{\Delta z}/_{\Delta t} \right) = (v_x, v_y, v_z)$

Speed $v = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Acceleration $\vec{a} = \lim_{\Delta t \to 0} \left({}^{\Delta v_x}/_{\Delta t}, {}^{\Delta v_y}/_{\Delta t}, {}^{\Delta v_z}/_{\Delta t} \right) = (a_x, a_y, a_z)$

Component of $\vec{a}$ parallel to $\vec{v}$: $a_{\parallel}$= rate of change of speed = $\Delta v / \Delta t$

Component of $\vec{a}$ perpendicular to $\vec{v}$: $a_{\perp}$. Affects only direction of $\vec{v}$.

Projectiles

When the only force is gravity (no air resistance, etc.), the horizontal (x) and vertical (y) components of the motion can be treated independently. For a projectile launched from (x_0, y_0) with initial speed v_0 at angle θ above horizontal, the initial velocity $\vec{v}_0 = (v_{0x}, v_{0y}) = (v_0 \cos\theta, v_0 \sin\theta)$, and

$$a_x = 0 \qquad\qquad a_y = -g$$

$$v_x = v_{0x} \qquad\qquad v_y = v_{0y} - gt$$

$$x = x_0 + v_{0x}t \qquad\qquad y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

These formulas require that the $+y$ direction be up. The magnitude of the acceleration due to gravity at Earth's surface is approximately 9.8 m/s^2, designated as g. Similar formulas apply for different coordinate systems.

Range Equation

The horizontal distance traveled by a projectile before landing at its launch height y_0 is $v_0{}^2 \sin 2\theta / g$.

Problems

There is not room on this worksheet for your work. Use scratch paper.

1. Three identical steel balls are released at the same time from the same height above the ground. One is released with initial speed 0 m/s, one with initial speed u upward, and one with initial speed u downward. Once released, all are in free-fall until they hit the ground.

   a. Draw a diagram of the initial situation. Show axis directions and the location of the origin.

   b. Construct, for each ball, the kinematic equation giving height as a function of time.

   c. Construct equations for the height differences as functions of time between:

      i. The ball initially moving upward and the ball released from rest.

      ii. The ball released from rest and the ball initially moving downward.

   d. Find the formula for the maximum height above the ground reached by the ball initially moving upward.

   e. Substitute the height found in part d into the height equations for

      i. the ball released from rest

      ii. the ball initially moving downward

      and solve for t to find when these balls reach that height. What do you find? What do these results mean?

2. A projectile is launched from height h at speed v_0 and angle θ above the horizontal and lands on the ground.

   a. Draw a diagram of the initial situation. Show axis directions and the location of the origin.

   b. Find the formula for the maximum height reached by the projectile.

   c. Find the formula for the time at which the projectile reaches the top of its arc.

   d. Find the formula for the projectile's velocity $\vec{v} = (v_x, v_y)$ at the top of its arc.

   e. Find the formula for the vertical component of the projectile's velocity just as it reaches the ground.

   f. Find the formula for the time at which the projectile lands on the ground.

   g. Find the formula for the horizontal distance the projectile travels to the top of its arc.

   h. Find the formula for the horizontal distance the projectile travels to land on the ground.

   i. The projectile is a car tire fired from a hobbyist's catapult at a speed of 20.0 m/s at an angle of 50° above horizontal from a height of 2.50 m above the ground. How far away horizontally from the launch does it land on the ground?