
Worksheet 3: Trajectory Kinematics

Objectives

- Represent position, velocity and acceleration as vectors.
- Work with ballistic trajectories in the vertical and horizontal directions.

Summary

3-D Position, velocity, and acceleration vectors

Position $\vec{r} = (x, y, z)$

Velocity $\vec{v} = \lim_{\Delta t \rightarrow 0} \left(\Delta x / \Delta t, \Delta y / \Delta t, \Delta z / \Delta t \right) = (v_x, v_y, v_z)$

Speed $v = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Acceleration $\vec{a} = \lim_{\Delta t \rightarrow 0} \left(\Delta v_x / \Delta t, \Delta v_y / \Delta t, \Delta v_z / \Delta t \right) = (a_x, a_y, a_z)$

Component of $\vec{a}$ parallel to $\vec{v}$: $a_{\parallel}$ = rate of change of speed $= \Delta v / \Delta t$

Component of $\vec{a}$ perpendicular to $\vec{v}$: $a_{\perp}$. Affects only direction of $\vec{v}$.

Projectiles

When the only force is gravity (no air resistance, etc.), the horizontal (x) and vertical (y) components of the motion can be treated independently. For a projectile launched from (x_0, y_0) with initial speed v_0 at angle θ above horizontal, the initial velocity $\vec{v}_0 = (v_{0x}, v_{0y}) = (v_0 \cos \theta, v_0 \sin \theta)$, and

$$a_x = 0$$

$$a_y = -g$$

$$v_x = v_{0x}$$

$$v_y = v_{0y} - gt$$

$$x = x_0 + v_{0x}t$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

These formulas require that the $+y$ direction be up. The magnitude of the acceleration due to gravity at Earth's surface is approximately 9.8 m/s^2 , designated as g . Similar formulas apply for different coordinate systems.

Range Equation

The horizontal distance traveled by a projectile before landing at its launch height y_0 is $v_0^2 \sin(2\theta) / g$.

Problems

There is not room on this worksheet for your work. Use scratch paper.

1. A projectile is launched from height h at speed v_0 and angle θ above the horizontal and lands on the ground.
 - a. Draw a diagram of the initial situation. Show axis directions and the location of the origin.
 - b. Find the formula for the maximum height reached by the projectile.
 - c. Find the formula for the time at which the projectile reaches the top of its arc.
 - d. Find the formula for the projectile's velocity $\vec{v} = (v_x, v_y)$ at the top of its arc.
 - e. Find the formula for the vertical component of the projectile's velocity just as it reaches the ground.
 - f. Find the formula for the time at which the projectile lands on the ground.
 - g. Find the formula for the horizontal distance the projectile travels to the top of its arc.
 - h. Find the formula for the horizontal distance the projectile travels to land on the ground.
 - i. The projectile is a car tire fired from a hobbyist's catapult at a speed of 20.0 m/s at an angle of 50° above horizontal from a height of 2.50 m above the ground. How far away horizontally from the launch does it land on the ground?

2. A physics student throws a rubber duck from an apartment balcony that is 30.0 meters above the ground. The duck strikes the ground 1.94 seconds later at a distance of 15.5 meters from the wall of the apartment building.
 - a. What was the initial velocity of the rubber duck?
 - b. If you have time, you can also find the velocity of the rubber duck just as it strikes the ground.