Discussion 4: Curved Paths and Forces

Objective

- Relate position, velocity, and acceleration vectors in uniform circular motion.

Summary

Uniform circular motion

An object traveling at a constant speed *v* in a circular path of radius *r* travels a distance $2\pi r$ to complete one circuit. If one circuit takes time *T* (the *period*), then $v = 2\pi r/T$. The *frequency* $f = 1/T$.

Radians

It is convenient to specify positions on a circular path as angles. The simplest way to specify an angle is as a portion of a circle. Once around a circle of radius *r* is a path length of $2\pi r$. When a particle travels a distance *s* around a circular path of radius *r*, the angle θ it completes can be expressed as $\theta = s/r$. The value s/r is the measure of the angle in *radians*. Radians are dimensionless, a ratio of distances. A complete circle is 2π radians.

Angular speed

The rate at which an object traverses a circular path at a constant *tangential* speed *v* can be conveniently expressed as *angular* speed ω, in radians/s. Angular speed ω and tangential speed *v* are related by the radius *r*: $v = \omega r$. Similarly, $\omega = 2\pi f$ and $T = 2\pi/\omega$.

Acceleration

A particle traveling in uniform circular motion experiences a centripetal acceleration with magnitude $a = v^2/r$ directly toward the center of the path. In terms of other quantities, $a = 4\pi^2 r/T^2 = 4\pi^2 f^2 r = \omega^2 r$.

Problems

There is not room on this worksheet for your work. Use your own scratch paper.

1. A carnival merry-go-round rotates about a vertical axis at a constant rate. A painted wooden horse on the merry-go-round has a constant speed of 1.50 m/s. The magnitude of its acceleration is 0.563 m/s^2.

   a. At one moment, the horse's acceleration is due east. Where is the horse: how far from the center and in what direction from the center of the carousel?

   b. What is the angular speed of the carousel?

c. What is the period of rotation of the carousel?

2. A particle travels at constant speed v in a circular path with radius R.

   a. The position of the particle can be expressed in polar coordinates as $\vec{R} = (R, \theta)$, where R is the distance from the origin and θ is the angle counterclockwise from the $+x$ axis. Find expressions for the Cartesian x- and y-coordinates $\vec{R} = (x. y)$ in terms of R and θ.

   b. The angle θ changes steadily with time: $\theta = \omega t$, where ω is a constant angular speed. Substitute ωt for θ in the expressions you obtained in part a.

   c. If the particle makes one complete revolution in time T, then its speed $v = 2\pi R/T$ and angular speed $\omega = 2\pi/T$. Use these formulas to find v in terms of ω.

3. A 1260-kg car makes a right turn in a segment of a circle of radius r = 4.2 m. (The paths of the right and left sides of the car will have different radii, but we'll ignore that by treating the car as a point. You can imagine what would need to change to make the model more realistic.)

   a. Draw a free body diagram for the car.

   b. What force or forces provides the car's (inward) acceleration?

   c. What is a reasonable coefficient of static friction between a car tire and wet pavement?

   d. Assuming the coefficient of static friction you chose in part c, what is the fastest speed that the car can take through the turn without slipping?