Discussion 5: Work and energy

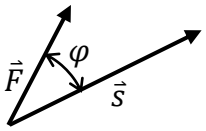
Summary

Work

Work is the effect of a force applied to an object while it undergoes a displacement. Both force $\vec{F}$ and displacement $\vec{s}$ are vectors; the work W done is their dot product, $W = \vec{F} \cdot \vec{s}$. The unit of work is the **joule**, J = kg·m^2/s^2.

Dot product

The dot product of two vectors is a measure of their geometric overlap. The dot product of two vectors is a *scalar* quantity. If the vectors are placed beside each other, showing the angle between them as φ, the dot product can be found by the formula $\vec{F} \cdot \vec{s} = Fs \cos \varphi$.



If the vectors are expressed as their Cartesian components, $\vec{F} = (F_x, F_y, F_z)$ and $\vec{s} = (s_x, s_y, s_z)$, their dot product is given by the formula $\vec{F} \cdot \vec{s} = F_x s_x + F_y s_y + F_z s_z$.

Kinetic energy

The kinetic energy of an object is the amount of work required to accelerate it from rest to its current speed. The kinetic energy of an object of mass m traveling at speed v is ½ mv^2.

Work-energy theorem

The kinetic energy change of a body after some displacement is equal to the total work done on it over that displacement.

Gravitational potential energy

To lift an object of mass m vertically upward a height h against a uniform gravitational field g requires doing work of mgh. This is the object's gravitational potential energy.

Power

Power is the rate of doing work, $P = W/\Delta t$. Like work and energy, power is scalar. To do work on something, it must be moving; the power applied by force $\vec{F}$ to an object moving with velocity $\vec{v}$ is $P = \vec{F} \cdot \vec{v}$. The unit of power is the **watt**, W = J/s = kg·m^2/s^3.

Problem

Barbara pulls her little sister Annie up a snow-covered hill on her sled. It takes them 200 seconds, a little over three minutes, to reach the top of the hill. Annie and the sled together have a mass 35.0 kg; the hill is 40.0 meters long, and the top of the hill is 4.0 meters higher than the bottom. The coefficient of kinetic friction between the snow and the sled is 0.060.

a. Annie reaches the top of the hill at rest. How much work was done on her and her sled along her ascent up the hill by each of the following forces?

   - Gravity

- The normal force
- Friction
- Barbara's pull

b. What is Barbara's average power as she pulls the sled up the hill?

c. When Annie is ready to ride down the hill, Barbara gives her sled a gentle nudge to start. On her descent down the hill, how much work is done on Annie and her sled by each of the following forces?

- Gravity
- Friction
- The normal force

d. What is Annie's kinetic energy when she reaches the bottom of the hill?

e. What is Annie's speed when she reaches the bottom of the hill?