Discussion 6: Work and energy

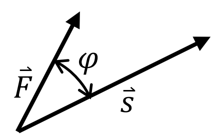
Summary

Work

Work is the effect of a force applied to an object while it undergoes a displacement. Both force $\vec{F}$ and displacement $\vec{s}$ are vectors; the work W done is their dot product, $W = \vec{F} \cdot \vec{s}$. The unit of work is the joule, J = kg·m^2/s^2.

Dot product

The dot product of two vectors is a measure of their geometric overlap. The dot product of two vectors is a *scalar* quantity. If the vectors are placed beside each other, showing the angle between them as φ, the dot product can be found by the formula $\vec{F} \cdot \vec{s} = Fs \cos \varphi$.



If the vectors are expressed as their Cartesian components, $\vec{F} = (F_x, F_y, F_z)$ and $\vec{s} = (s_x, s_y, s_z)$, their dot product is given by the formula $\vec{F} \cdot \vec{s} = F_x s_x + F_y s_y + F_z s_z$.

Kinetic energy

The kinetic energy of an object is the amount of work required to accelerate it from rest to its current speed. The kinetic energy of an object of mass m traveling at speed v is ½ mv^2.

Work-energy theorem

The kinetic energy change of a body after some displacement is equal to the total work done on it over that displacement.

Gravitational potential energy

To lift an object of mass m vertically upward a height h against a uniform gravitational field g requires doing work of mgh. This is the object's gravitational potential energy.

Hooke's law

The elasticity of springs and all solid objects can be approximately modeled by $\vec{F} = -k\vec{x}$, where $\vec{F}$ is the force exerted by the spring, $\vec{x}$ is its distortion from its un-stressed size (length), and k is a constant characteristic of the spring. The negative sign in the formula indicates that the spring exerts a fore to return to its un-stressed size.

The potential energy of a stretched or compressed spring is ½ kx^2.

Newton's gravitational formula

The gravitational attraction between two point masses has a magnitude $F = Gm_1m_2/r^2$, where G is the universal gravitational constant 6.674 × 10–11 N·m^2/kg^2, m_1 and m_2 are the masses of the objects, and r is the distance between them.

The gravitational potential energy of two objects is $-Gm_1m_2/r$.

Problems

1. A 108-kg satellite in low earth orbit travels in a circular path 500.0 km above the Earth's equator. The equatorial radius of the Earth is 6378 km, and the mass of the Earth is 5.97×10^{24} kg.

   a. What is the radius of the satellite's orbit?

   b. The force accelerating the satellite toward the center of its path is the force of its gravitational attraction to the Earth.

   $$m_2 \frac{v^2}{r} = G \frac{m_1 m_2}{r^2}$$

   Algebraically solve this expression to find the formula for the tangential speed v of the satellite in terms of the other quantities.

   c. What is the satellite's speed in meters per second?

   d. What is the satellite's kinetic energy?

   e. What is the satellite's gravitational potential energy?

   f. The satellite's speed is $2\pi r/T$, where T is its orbital period, the time to make one complete orbit. What is this satellite's orbital period in minutes?

2. Barbara pulls her little sister Annie up a snow-covered hill on her sled. Annie and the sled together have a mass 35.0 kg; the hill is 40.0 meters long, and the top of the hill is 4.00 meters higher than the bottom. The coefficient of kinetic friction between the snow and the sled is 0.060.

   a. Annie reaches the top of the hill at rest. How much work was done on her and her sled along her ascent up the hill by each of the following forces?

      - Gravity
      - The normal force
      - Friction
      - Barbara's pull

   b. When Annie is ready to ride down the hill, Barbara gives her sled a gentle nudge to start. On her descent down the hill, how much work is done on Annie and her sled by each of the following forces?

      - Gravity
      - Friction
      - The normal force

   c. What is Annie's kinetic energy when she reaches the bottom of the hill?

   d. What is Annie's speed when she reaches the bottom of the hill?