Discussion 7: Conservation of energy

Summary

Mechanical Energy

Mechanical energy comprises the forms of energy that are readily converted to work: kinetic energy, and potential energy of all kinds.

Conservation of Mechanical Energy

Mechanical energy is conserved in any process that transfers energy between different forms of mechanical energy, such as from kinetic energy to potential energy, or between different types of potential energy.

If we know that mechanical energy is conserved in some process, we know that throughout the process the total mechanical energy remains constant. For instance, if a process involves two configurations, 1 and 2, each with its associated kinetic energy K, gravitational potential energy U_g, and elastic potential energy U_e,

$$E_1 = E_2$$

$$K_1 + U_{g1} + U_{e1} = K_2 + U_{g2} + U_{e2}$$

$$\tfrac{1}{2}\,mv_1^2 + mgy_1 + \tfrac{1}{2}\,kx_1^2 = \tfrac{1}{2}\,mv_2^2 + mgy_2 + \tfrac{1}{2}\,kx_2^2$$

Conservative and non-conservative forces

If only conservative forces act on the objects in a system of interest throughout a process, mechanical energy is conserved. **Conservative** forces are forces associated with a potential energy; examples include gravity and the force exerted by a Hooke's law spring.

Non-conservative forces, on the other hand, do not conserve mechanical energy. Examples of non-conservative forces include friction and the force provided by a motor.

Conservation of energy

When non-conservative forces act, we can account for their effect: the work W_N done by a non-conservative force changes the system's mechanical energy.

$$E_1 + W_N = E_2$$

Problems

1. A 42-kg snowman slides along frictionless ice at a speed of 2.10 m/s toward a coil spring with a spring constant of 1700 N/m, as 1. The snowman runs into the spring, compressing the spring until it momentarily stops the snowman, as 2. We want to find how far the spring compresses.

a. The only horizontal force, from the spring, is conservative, so we can set up a conservation of energy equation.

$$E_1 = E_2$$

$$½\, mv_1^2 + ½\, kx_1^2 = ½\, mv_2^2 + ½\, kx_2^2$$

b. The problem requires that $x_1 = 0$ and $v_2 = 0$. Solve the simplified equation for x_2.

c. Mathematically, there are two solutions for x_2 that satisfy the conservation of energy equation.

   i. Which one is the one you want?

   ii. What does the other solution mean?

2. Industrious students construct a potato cannon. In its first trial on Fraternity Mall, the cannon fires a 0.450-kg potato at a speed of 24.5 m/s at an angle of 35 degrees above horizontal. We want to find out how high above the launch point the potato rises in its arc, and how fast it is traveling at that point. We could use kinematics to find these quantities, but we'll use a combination of kinematics and conservation of energy instead. We'll call state 1 right after the potato leaves the muzzle of the cannon, and state 2 the top of the potato's arc. We'll also assume, as usual, that the non-conservative force of drag is not significant, leaving only the conservative force of gravity to consider.

   a. First, the kinematics. Find the horizontal and vertical components of the potato's initial velocity (state 1).

   b. What is the potato's kinetic energy at state 1?

   c. We ought to know that at the top of the potato's arc (state 2), its speed *v* is equal to its initial horizontal component of velocity. What it the potato's kinetic energy at state 2?

   d. Here's the magic. We assume that the potato's mechanical energy is the same at states 1 and 2.

$$E_1 = E_2$$

$$K_1 + U_{g1} = K_2 + U_{g2}$$

$$K_1 + mgy_1 = K_2 + mgy_2$$

$$K_1 - K_2 = mgy_2 - mgy_1$$

$$K_1 - K_2 = mg(y_2 - y_1)$$

   You can solve this the rest of the way to find the greatest height $y_2 - y_1$.

   e. You already know the speed at the top of the arc. You've known since step a.

3. A 70-kilogram base runner runs toward home plate at a speed of 7.20 meters per second. To elude the tag, he begins to slide when he is a distance of 3.70 meters from the plate. His coefficient of kinetic friction with the dirt is 0.65. We want to find how fast the runner is sliding when he reaches the plate.

   We'll call state 1 when the runner begins to slide and state 2 when he reaches the plate. The work done on the runner by friction is non-conservative.

$$E_1 + W_N = E_2$$
$$K_1 + W_N = K_2$$

   a. What is the formula for the force of friction on the sliding runner?

   b. What is the formula for the work done by friction as the runner slides to the plate?

   c. Substitute the appropriate formulas for K_1, W_N, and K_2 into the conservation of energy equation.

   d. Solve the conservation of energy equation for speed v_2.

   e. If you are ambitious, you can also find how far past the plate the runner slides.

4. Annie rides down a snow-covered hill on her sled. Annie and the sled together have a mass 35.0 kg; the hill is 40.0 meters long, and the top of the hill is 4.0 meters higher than the bottom. The coefficient of kinetic friction between the snow and the sled is 0.060. At the bottom of the hill, the ground is level.

   a. How fast is Annie moving when she reaches the bottom of the hill?

   b. How far does Annie coast on the level ground at the bottom of the hill before coming to a stop?

5. A boy sits at rest at the top of a frictionless slide, a height H above the ground. With a negligible push, he begins sliding downward. The end of the slide is a height h above the ground, and it launches the boy off the end horizontally. What horizontal distance d does the boy travel before landing on the ground?