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Discussion 9: Rigid-body Rotation

Summary

Rigid bodies

Rigid bodies do not change shape, size, or distribution of mass during a process. Their motion can always be described as a combination of translation of the center of mass and rotation about the center of mass.

Rotational kinematics

Radians

The extent of rotation about an axis is most naturally described in terms of **radians**, a dimensionless unit defined as movement of a point through a path length equal to its distance from the axis of rotation. The measure of any angle in radians is $\theta = s/r$, where s is the arc length of the path and r is the distance from the axis.

Velocity and acceleration

Angular velocity $\omega = \Delta\theta/\Delta t = v/r$, where $v = \Delta s/\Delta t$

Angular acceleration $\alpha = \Delta\omega/\Delta t = a_{\parallel}/r$, where $a_{\parallel}$ is the component of acceleration tangent to the path

Constant-α motion

$\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$

Torque

The angular analogue of force is **torque**, an outside influence that changes an object's rotation.

Torque $\tau = \vec{r} \times \vec{F}$, where $\vec{F}$ is a force applied to a body and $\vec{r}$ is a vector from the reference point to where the force is applied. $\tau = rF \sin\theta$, where θ is the angle between vectors $\vec{r}$ and $\vec{F}$. On a static body, all torques, like all forces, add to zero.

Rotational inertia

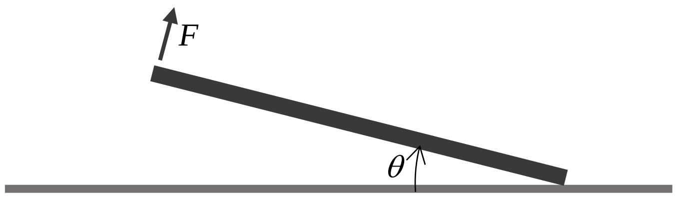
The tendency of a body to resist angular acceleration by an external torque is its rotational inertia or **moment of inertia** I. The moment of inertia of a point of mass m a distance r from the rotational axis is mr^2. The moment of inertia of a rigid body that is not a point depends on its mass and the distribution of its mass about its center. Some formulas are in your textbook's Table 9.1.

Newton's second law

Newton's second law for rotational motion is $\alpha = \Sigma\tau/I$.

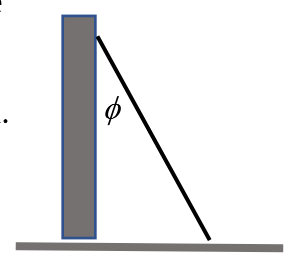
Problems

1. A worker removes a manhole cover with a mass of m = 54 kg and diameter D = 0.96 m by lifting one edge while the opposite edge remains in its frame. We would like to find the magnitude of perpendicular force $\vec{F}$ needed to hold the cover as its angle θ from the ground changes.



   a. Consider the torque about the ground contact point caused by the force $\vec{F}$.
      i. What is the distance from the contact point to where the force is applied?
      ii. What is the angle between the force vector and the radius vector?
      iii. Is the torque positive (counterclockwise) or negative (clockwise)?
      iv. Write the formula for this torque.
   b. Consider the torque about the ground contact point caused by the weight $m\vec{g}$ of the cover.
      i. We consider the weight of an object to be applied at its center of mass, which in a uniform gravitational field is its center of mass. What is the distance from the contact point to the center of mass?
      ii. What is the angle between the force vector $m\vec{g}$ and the radius vector?
      iii. Is the torque positive (counterclockwise) or negative (clockwise)?
      iv. Write the formula for this torque.
   c. To hold the manhole cover steady at angle θ, the net torque on it must be zero. Use this fact to find the formula for the magnitude F when the cover is tilted at angle θ.

2. A ladder rests against a wall at angle ϕ from vertical. The mass of the ladder is m and its length is L. The ladder's coefficient of static friction with the ground is μ_s, while the wall it leans against is frictionless. We are concerned about the ladder sliding down the wall.



   There are several ways to consider this problem. We will evaluate all torques about the point where the bottom of the ladder contacts the ground. Other reference points can be used as well; another intuitive point is the ladder's center of mass.

   a. There are four forces acting on the ladder.
      1. Its weight $\vec{w} = m\vec{g}$
      2. Normal force from the ground $\vec{N}_1$
      3. Normal force from the wall $\vec{N}_2$

4. Friction against the ground $f = \mu_s N_1$

i. Draw a free body diagram for the ladder showing these forces.

ii. When the ladder is static, these forces add to zero in both the horizontal and vertical directions. Forces $\vec{w}$ and $\vec{N}_1$ are purely vertical, while $\vec{f}$ and $\vec{N}_2$ are purely horizontal. Thus, we know that $N_1 = w = mg$. At the limit of static friction, $f = \mu_s mg$. Balancing forces requires $\vec{N}_2 = f$.

iii. Write out the *x*- and *y*-components of these four forces.

b. Now we need to find the torques resulting from these forces.

i. The torques resulting from forces $\vec{N}_1$ and $\vec{f}$ are both zero. Why? ________________

ii. Find the magnitude of the torque produced by the weight of the ladder. Is this torque positive (counterclockwise) or negative (clockwise)?

iii. Find the magnitude of the torque produced by the normal force $\vec{N}_2$. Is this torque positive (counterclockwise) or negative (clockwise)?

c. For the ladder to rest at angle ϕ, the net torque on it must be zero. Use this fact to find the formula for the coefficient of friction μ_s in terms of the angle ϕ.

3. A uniform cylinder with a mass of M = 20.0 kg and radius R = 0.200 m is initially spinning about an axle with a rotational speed of $\omega_0 = 10.0$ rad/s.

a. What is the moment of inertia *I* of the cylinder?

A brake pad presses against the face (circular end) of the cylinder at a distance of *d* = 0.150 m from the center with force *F* = 15.0 N. The coefficient of kinetic friction between the brake pad and the cylinder is $\mu_k = 0.400$. This creates a force of friction against the cylinder's rotation.

b. What is the magnitude *f* of the force of friction?

c. What is the magnitude of the torque on the cylinder created by the frictional force?

d. What is the angular acceleration of the cylinder?

e. How far does the cylinder turn as it comes to a stop:

i. in radians?

ii. in revolutions?

f. How much time does the cylinder take to stop?