
Discussion 9: Torque

Summary

Torque

The angular analogue of force is **torque**, an outside influence that changes an object's rotation.

Torque $\tau = \vec{r} \times \vec{F}$, where $\vec{F}$ is a force applied to a body and $\vec{r}$ is a vector from the reference point to where the force is applied. $\tau = rF \sin \theta$, where θ is the angle between vectors $\vec{r}$ and $\vec{F}$. On a static body, all torques, like all forces, add to zero.

Rotational inertia

The tendency of a body to resist angular acceleration by an external torque is its rotational inertia or **moment of inertia** I . The moment of inertia of a point of mass m a distance r from the rotational axis is mr^2 . The moment of inertia of a rigid body that is not a point depends on its mass and the distribution of its mass about its center. Some formulas are in your textbook's Table 9.1.

Newton's second law

Newton's second law for rotational motion is $\alpha = \Sigma \tau / I$.

Center of mass

The center of mass of a body comprising N point masses (fundamentally, all bodies are composed of point masses), each having mass m_i and located at position x_i , is

$$x_{\text{cm}} = \frac{\sum_{i=1}^N x_i m_i}{\sum_{i=1}^N m_i}$$

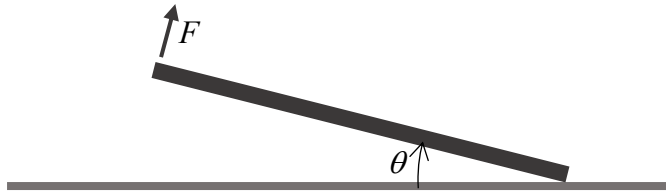
To find the center of mass of a system containing several extended bodies, treat each of the component bodies as a point mass located at its center of mass

If such a system is isolated (receives zero net force from outside the system), conservation of momentum requires that the velocity of its center of mass is constant.

Problems

1. A 45.0-kg woman stands up in a 60.0-kg canoe that is 5.00 m long. She walks from a point 0.50 m from one end to a point 0.50 m from the other end. Neglect drag from the water on the canoe.
 - A. Relative to solid ground, how far does the canoe move during this process? Assume that the center of mass of (canoe + woman) does not move.
 - B. Relative to solid ground, how far does the woman move during this process? How far does the canoe move?

2. A worker removes a manhole cover with a mass of $m = 54 \text{ kg}$ and diameter $D = 0.96 \text{ m}$ by lifting one edge while the opposite edge remains in its frame. We would like to find the magnitude of perpendicular force $\vec{F}$ needed to hold the cover as its angle θ from the ground changes.



- A. Consider the torque about the ground contact point caused by the force $\vec{F}$.
- What is the distance from the contact point to where the force is applied?
 - What is the angle between the force vector and the radius vector?
 - Is the torque positive (counterclockwise) or negative (clockwise)?
 - Write the formula for this torque.
- B. Consider the torque about the ground contact point caused by the weight $m\vec{g}$ of the cover.
- We consider the weight of an object to be applied at its center of mass, which in a uniform gravitational field is its center of mass. What is the distance from the contact point to the center of mass?
 - What is the angle between the force vector $m\vec{g}$ and the radius vector?
 - Is the torque positive (counterclockwise) or negative (clockwise)?
 - Write the formula for this torque.
- C. To hold the manhole cover steady at angle θ , the net torque on it must be zero. Use this fact to find the formula for the magnitude F when the cover is tilted at angle θ .
3. A uniform cylinder with a mass of $M = 20.0 \text{ kg}$ and radius $R = 0.200 \text{ m}$ is initially spinning about an axle with a rotational speed of $\omega_0 = 10.0 \text{ rad/s}$.
- A. What is the moment of inertia I of the cylinder?
- A brake pad presses against the rim (tubular surface) of the cylinder with force $F = 15.0 \text{ N}$. The coefficient of kinetic friction between the brake pad and the cylinder is $\mu_k = 0.400$. This creates a force of friction against the cylinder's rotation.
- B. What is the magnitude f of the force of friction?
- C. What is the magnitude of the torque on the cylinder created by the frictional force?
- D. What is the angular acceleration of the cylinder?
- E. How far does the cylinder turn as it comes to a stop:
- in radians?
 - in revolutions?
- F. How much time does the cylinder take to stop?