Discussion 10: Angular Momentum and Oscillations

Summary

Rotational work and energy

Kinetic energy

The rotational kinetic energy of a rotating body is $K_r = ½\, I\omega^2$.

The motion of any rigid body can be described as a combination of translation of its center of mass and rotation about its center of mass. Its total kinetic energy is
$K = K_t + K_r = ½\, \mathrm{m}v^2 + ½\, I\omega^2$.

Work

As a force $\vec{F}$ acting along a displacement $\vec{s}$ does work $\vec{F} \cdot \vec{s}$, a torque $\vec{\tau}$ acting along an angular displacement $\vec{\theta}$ does work $\vec{\tau} \cdot \vec{\theta}$. This allows an interpretation of the unit for torque, N·m, as equivalent to a J/rad.

Angular momentum

Angular momentum of a point mass with respect to a reference point is $\vec{L} = \vec{r} \times \vec{p}$, where $\vec{r}$ is the vector from the reference point to the point mass, and $\vec{p}$ is the point mass's momentum.

The angular momentum of a rigid rotor is $I\vec{\omega}$, with *I* being the body's moment of inertia for rotation about the reference point.

The torque applied to a body is the rate of change of its angular momentum, $\vec{\tau} = \Delta\vec{L}/\Delta t$.

Conservation of angular momentum

If a body receives no external torques, its angular momentum does no change.

If its angular momentum changes, its angular velocity changes inversely. $I_1\vec{\omega}_1 = I_2\vec{\omega}_2$

Simple harmonic motion

An object of mass m away from equilibrium acted on only by a restoring Hooke's law force $F = -kx$ moves in simple harmonic motion: oscillations obeying the kinematic equations

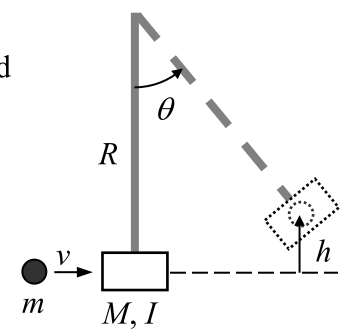
$x = A\cos(\omega t)$ $\qquad v = -A\omega\sin(\omega t)$ $\qquad a = -A\omega^2\cos(\omega t)$,

where the parameter $\omega = \sqrt{k/m}$. This is known as the angular frequency. It is related to the frequency f and period T of the oscillation by $f = \omega/2\pi$ and $T = 1/f = 2\pi/\omega$.

Because the Hooke's law force is conservative, mechanical energy is conserved throughout the oscillation. $E = K + U_e = ½\, mv^2 + ½\, kx^2 = ½\, kA^2$.

Problems

1. A *ballistic pendulum* is a venerable device used to indirectly measure cannon muzzle velocities. The cannonball is fired horizontally and is immediately caught and held by a pivoted catcher assembly. The catcher swings upward, allowing the initial cannonball velocity to be deduced.

   The ball of mass m is fired at speed v and caught at the center of mass of a pivoted catcher of mass M, hanging a distance R below its frictionless pivot. The catcher has a moment of inertia I about the pivot. The catcher swings to a maximum angle of θ from the vertical, raising its center of mass by h.

   a. Use conservation of angular momentum to find the angular speed ω of the assembly immediately after catching the cannonball. (Treat the cannonball as a point particle.)

   b. Use conservation of mechanical energy to find the maximum height h attained by the catcher.

   c. What is the maximum angle θ attained by the catcher?

   d. Let's try it with some numbers. The cannonball's mass is 5.00 kg, its initial speed is 200. m/s, the mass of the catcher is 200.0 kg, the catcher is 5.00 m from the pivot, and the moment of inertia of the catcher is 5020 kgm^2.

      i. What is the angular speed of the assembly after the catch?

      ii. How high above its starting height does the catcher swing?

      iii. What is the maximum angle θ attained by the catcher?

2. A 42-kg snowman slides along frictionless ice at a speed of 2.10 m/s toward a coil spring with a spring constant of 1700 N/m, as 1. The snowman runs into the spring, latching onto it.

   a. Knowing that mechanical energy is conserved, what is the maximum *compression* of the spring?

   b. What is the *amplitude* of the snowman's oscillation?

   c. What is the *period* of the snowman's oscillation?

   d. What is the maximum *speed* of the snowman in its oscillation?

   e. What is the maximum *acceleration* of the snowman in its oscillation?

   f. What is the maximum *net force* acting on the snowman in its oscillation?