Discussion 10: Torque and angular momentum

Summary

Torque

The angular analogue of force is **torque**, an outside influence that changes an object's rotation.

Torque $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{F}$ is a force applied to a body and $\vec{r}$ is a vector from the reference point to where the force is applied. $\tau = rF \sin\theta$, where θ is the angle from $\vec{r}$ to $\vec{F}$. On a static body, all torques, like all forces, add to zero.

Rotational inertia

The tendency of a body to resist angular acceleration by an external torque is its **moment of inertia** *I*. The moment of inertia of a point of mass *m* a distance *r* from the rotational axis is mr^2. The moment of inertia of a rigid body that is not a point depends on its mass and the distribution of its mass about the axis of rotation. Some formulas are in your textbook's Table 9.1.

Newton's second law

Newton's second law for rotational motion is $\alpha = \Sigma\tau / I$.

Rotational work and energy

Kinetic energy

The rotational kinetic energy of a rotating body is $K_r = ½\, I\omega^2$.

The motion of any rigid body can be described as a combination of translation of its center of mass and rotation about its center of mass. Its total kinetic energy is $K = K_t + K_r = ½\, \mathrm{m}v^2 + ½\, I\omega^2$.

Work

As a force $\vec{F}$ acting along a displacement $\vec{s}$ does work $\vec{F} \cdot \vec{s}$, a torque $\vec{\tau}$ acting along an angular displacement $\vec{\theta}$ does work $\vec{\tau} \cdot \vec{\theta}$. This allows an interpretation of the unit for torque, N·m, as equivalent to a J/rad.

Angular momentum

Angular momentum of a point mass with respect to a reference point is $\vec{L} = \vec{r} \times \vec{p}$, where $\vec{r}$ is the vector from the reference point to the point mass, and $\vec{p}$ is the point mass's momentum.

The angular momentum of a rigid rotor is $I\vec{\omega}$, with *I* being the body's moment of inertia for rotation about the reference point.

The torque applied to a body is the rate of change of its angular momentum, $\vec{\tau} = \Delta\vec{L} / \Delta t$.

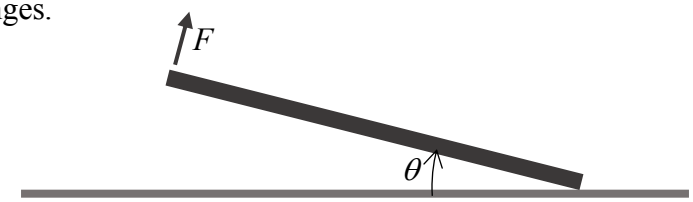
Conservation of angular momentum

If a body receives no net external torque, its angular momentum does not change.

If its moment of inertia changes, its angular velocity changes inversely. $I_1\vec{\omega}_1 = I_2\vec{\omega}_2$.

Problems

1. A worker removes a manhole cover with a mass of m = 54 kg and diameter D = 0.96 m by lifting one edge while the opposite edge remains in its frame. We would like to find the magnitude of perpendicular force $\vec{F}$ needed to hold the cover as the cover's angle θ from the ground changes.



   A. Consider the torque about the ground contact point caused by the force $\vec{F}$.
      i. What is the distance from the contact point to where the force is applied?
      ii. What is the angle between the force vector and the radius vector?
      iii. Is the torque positive (counterclockwise) or negative (clockwise)?
      iv. Write the formula for this torque.
   B. Consider the torque about the ground contact point caused by the weight $m\vec{g}$ of the cover.
      i. We consider the weight of an object to be applied at its center of gravity, which in a uniform gravitational field is its center of mass. What is the distance from the contact point to the center of gravity?
      ii. What is the angle between the force vector $m\vec{g}$ and the radius vector?
      iii. Is the torque positive (counterclockwise) or negative (clockwise)?
      iv. Write the formula for this torque.
   C. To hold the manhole cover steady at angle θ, the net torque on it must be zero. Use this fact to find the formula for the magnitude F when the cover is tilted at angle θ.

2. In the past, before child safety was as high a priority as it is now, many playgrounds contained carousels. (You can watch a video of the carousel that used to stand in Laramie's Harbon Park at https://youtu.be/BFBGr0GsAPY.) These were circular horizontal platforms that would rotate about their centers. They were usually equipped with rails to make them easy to push, and to help riders hold on when the carousels spun rapidly.
   A. Suppose a playground carousel has a radius of 1.70 meters and a mass of 200. kilograms. We can approximate it as a uniform cylinder to estimate its moment of inertia. (The formula for the moment of inertia of a uniform cylinder rotating about its principal axis is ½ MR^2.) What is the approximate moment of inertia of the carousel?
   B. Carousels weren't built so that we could watch them spin—they were made to ride. Suppose four 30.-kilogram children ride on the carousel. They are evenly spaced around its rim. (OK, not quite at the rim, because then they'd fall off. Let's say they stand 0.15

meters from the rim, at 1.55 meters from the axis.) Approximate the children as point masses. What amount do the children add to the moment of inertia of the carousel?

There's no point in just standing on a carousel—ride it while it spins. Suppose that the carousel with the four riders rotates makes one complete circle every 3.5 seconds.

C. What is the angular momentum of the rotating carousel (including riders)?

D. What is the kinetic energy of the rotating carousel (including riders)?

E. What is the centripetal acceleration of the riders?

Suppose that the riders move together toward the center of the carousel, to a distance of 0.90 meters from the axis.

F. Is the angular momentum of the carousel (including riders) conserved?

G. What is the new period of rotation of the carousel?

H. What is the new kinetic energy of the rotating carousel (including riders)?

I. What is the new centripetal acceleration of the riders?