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Discussion 11: Waves

Summary

Simple pendulum

Angular frequency $\omega = \sqrt{g/L}$, where g is the gravitational field strength and L is the length of the pendulum. Period $T = 2\pi/\omega$ and frequency $f = \omega/2\pi$. The period and frequency depend on the gravitational field and length of the pendulum, but do not depend on mass or amplitude.

Small angle approximation

The properties of a simple pendulum above are only approximate, and approach being strictly true as the amplitude of its oscillation approaches zero. In that limit, $\sin\theta = \theta$, with θ in radians.

Waves

A wave is an organized disturbance that propagates in time and space. In a **transverse** wave, the medium oscillates in a direction that is perpendicular to the propagation direction of the wave; in a **longitudinal** wave, the medium oscillates in the same direction that the wave propagates.

The simplest mathematical formula to describe wave motion is a sinusoidal function

$$y = A\cos(kx - \omega t + \varphi)$$

where A is the **amplitude** of the wave, $k = 2\pi/T$ is the **wave number**, $\omega = 2\pi/\lambda$ is the **angular frequency**, and φ is the **phase offset**. Here, T is the **period**, or repeat time of the wave; and λ is the **wavelength**, or repeat distance of the wave. The **propagation speed** of the wave is given by $u = \lambda/T = \omega/k$.

Transverse waves in a cord

The propagation speed of a wave in a cord of length density (mass per unit length) μ under tension F is $u = \sqrt{F/\mu}$.

Problems

1. A simple pendulum is constructed in a science museum with a high ceiling. The pendulum has a period of oscillation of 8.00 seconds.
   a. What is the length of the pendulum?
   b. When the pendulum travels a horizontal distance of 4.00 meters from one extreme of its swing to the other, what is the amplitude of its oscillation in radians?
   c. How good is the small angle approximation for this system? In other words, is the sine of the amplitude close to the value of the amplitude in radians?
   d. At this amplitude, what is the difference in height of the bob at the bottom of its swing and at an edge of its swing?

2. The lowest note on a grand piano has a frequency of 27.5 Hz. The string is 2 m (more or less; let's just assume it is exactly 2.000 m) long, and its tension is 1000 N (again, more or less, so let's say it's exactly 1000 N).

   a. What is the wavelength of the wave in the string?

   b. What is the propagation speed of the wave in the string?

   c. What must be the length density μ (mass per unit length, in units of kg/m or g/m) of the string?

   d. What must the mass of the string be?

   e. If the string is to be made of steel, which has a volume density ρ = 7800 kg/m^3, what must the volume of the string be?

   f. The volume of a cylinder of length L and diameter d is $V = \pi d^2 L/4$. What must the diameter of the string be?

   g. Is this "string" likely to flex like a string?

   h. How are the low strings on a piano actually constructed to give them the proper length densities?