Discussion 10: Angular Momentum and Oscillations

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In the interest of getting these solutions out before the exam, I have opted not to draw as many pictures as I normally would. I still encourage you to draw and label pictures before approaching a problem! I promise it helps.

1. A simple pendulum is constructed in a science museum with a high ceiling. The pendulum has a period of oscillation of 8.00 seconds.

1a) What is the length of the pendulum?

Our equation sheet gives us a way to relate the angular frequency to the weight of a pendulum on Earth:

$$\omega = \sqrt{\frac{g}{L}}. \tag{1}$$

The problem now is that we don't immediately know the angular frequency. Luckily, we have a method of finding the angular frequency from the period:

$$T = \frac{2\pi}{\omega}, \tag{2}$$

Or,

$$\omega = \frac{2\pi}{T}. \tag{3}$$

Putting this in for ω in our first equation:

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}}. \tag{4}$$

Now we can solve for L. Squaring both sides:

$$\left(\frac{2\pi}{T}\right)^2 = \frac{g}{L}. \tag{5}$$

Getting the L by itself:

$$L = g\left(\frac{T}{2\pi}\right)^2. \tag{6}$$

Now we can put in numbers:

$$L = (9.8)\left(\frac{(8)}{2\pi}\right)^2 = 15.9 \text{ m}. \tag{7}$$

1b) When the pendulum travels a horizontal distance of 4.00 meters from one extreme of its swing to the other, what is the amplitude of its oscillation in radians?

If we take the horizontal displacement of the bob to be 4.00, then we have two parts of a right triangle:

We know the side *opposite* the angle as well as the *hypotenuse*. Opposite over hypotenuse means sine! Now we know that:

$$\sin(\theta) = \frac{4.00}{15.9}. \tag{8}$$

To solve for θ, we take the inverse sine. Therefore,

$$\theta = \sin^{-1}\left(\frac{4.00}{15.9}\right) = 0.2545 \text{ rad}. \tag{9}$$

1c) How good is the small angle approximation for this system? In other words, is the sine of the amplitude close to the value of the amplitude in radians?

The small angle approximation says that:

$$\sin\theta \approx \theta. \tag{10}$$

We know from before that $\sin\theta = \frac{4.00}{15.9} = 0.2518$. We also found before that $\theta = 0.2545$. To quantify how close these values are to each other, we can use the percent difference equation:

$$\% \text{ difference between } x_1 \text{ and } x_2 = \frac{x_1 - x_2}{x_1} \times 100. \tag{11}$$

Therefore:

$$\% \text{ difference between } \theta \text{ and } \sin\theta = \frac{0.2545 - 2518}{2545} \times 100 = 1.06\%. \tag{12}$$

A percent difference of slightly over 1% tells us that this is definitely a safe time to use the small angle approximation.

1d) At this amplitude, what is the difference in height of the bob at the bottom of its swing and at an edge of its swing?

2. The lowest note on a grand piano has a frequency of 27.5 Hz. The string is 2 m (more or less; let's just assume it is exactly 2.000 m) long, and its tension is 1000 N (again, more or less, so let's say it's exactly 1000 N).

2a) What is the wavelength of the wave in the string?

We know from class that the lowest note playable on a string is also known as the *fundamental mode*. For the fundamental mode, FM, the wavelength is twice the length of the string:

$$\lambda_{FM} = 2L. \tag{13}$$

In our case:

$$\lambda_{FM} = 2(2) = 4.00 \text{ m}. \tag{14}$$

2b) What is the propagation speed of the wave in the string?

The speed u of any wave is related to its wavelength λ and frequency f by:

$$u = \lambda f. \tag{15}$$

For the fundamental mode:

$$u_{FM} = \lambda_{FM} f_{FM} \tag{16}$$

With our numbers:

$$u_{FM} = (4.00)(27.5) = 110 \text{ m/s}. \tag{17}$$

2c) What must the length density μ (mass per unit length, in units of kg/m or g/m) of the string?

Our equation sheet tells us that:

$$u = \sqrt{\frac{F}{\mu}}, \tag{18}$$

Where F is the tension of the cord, and μ is the length density. We can square both sides to get:

$$u^2 = \frac{F}{\mu}. \tag{19}$$

Solving for μ,

$$\mu = \frac{F}{u^2}. \tag{20}$$

Now we can put in numbers:

$$\mu = \frac{(1000)}{(110)^2} = 0.0826 \text{ kg/m}. \tag{21}$$

2d) What must the mass of the string be?

We know that the mass per meter is 0.0826 kg. Now we multiply by the length to get the total mass:

$$m \text{ [kg]} = \mu \left[\frac{\text{kg}}{\text{m}}\right] L \text{ [m]}. \tag{22}$$

With numbers:

$$m = (0.0826)(2) = 0.1653 \text{ kg}. \tag{23}$$

2e) If the string is to be made of steel, which has a volume density $\rho = 7800$ kg/m^3, what must the volume of the string be?

Density is mass per unit volume:

$$\rho = \frac{m}{V}. \tag{24}$$

We know ρ and m, so we can solve this equation for V to get:

$$V = \frac{m}{\rho}. \tag{25}$$

With numbers:

$$V = \frac{0.1653}{7800} = 21.19 \cdot 10^{-6} \text{ m}^3. \tag{26}$$

Converting from m^3 to cm^3:

$$V = \frac{21.19 \cdot 10^{-6} \text{ m}^3}{1} \left(\frac{100 \text{ cm}}{\text{m}}\right)^3 = 21.19 \cdot 10^{-6} \text{ m}^3 \cdot \frac{10^6 \text{ cm}^3}{\text{m}^3} = 21.19 \text{ cm}^3. \tag{27}$$

2f) The volume of a cylinder of length L and diameter d is $V = \pi d^2 L/4$. What must the diameter of the string be?

We have:

$$V = \frac{\pi d^2 L}{4}. \tag{28}$$

We can divide both sides by $\frac{\pi L}{4}$ to get:

$$d^2 = \frac{4V}{\pi L}. \tag{29}$$

Taking the square root:

$$d = \sqrt{\frac{4V}{\pi L}}. \tag{30}$$

Now we can put in numbers:

$$d = \sqrt{\frac{4(21.19 \cdot 10^{-6}\ \mathrm{m}^3)}{\pi(2)}} = 0.00367\ \mathrm{m}. \tag{31}$$

Knowing that there are 1000 mm in a m:

$$d = 3.67\ \mathrm{mm}. \tag{32}$$

2g) Is this ”string” likely to flex like a string?

If we look at a ruler, we can see that 3.67 mm is actually fairly thick as far as strings go. That, in combination with the fact that the string is made of steel, means that this is more like a steel rod than an actual string.

2h) How are the low strings on a piano actually constructed to give them the proper length densities?

As it turns out, strings on string instruments are made not of single strands, but of multiple thin strands wrapped around each other to get the length density and diameters needed.