Worksheet 12: Gravity

Newton's gravitational formula

The gravitational force exerted on particle 1 by particle 2 is $F = G\frac{m_1 m_2}{r^2}$, where G = gravitational constant = 6.67×10^{-11} Nm^2/kg^2; m_1, m_2 are the masses of the particles; and r is the distance between the particles.

Gravitational potential energy

$U = -G\frac{m_1 m_2}{r}$. Note that U increases with separation r, up to a maximum of zero.

Circular orbit

In a circular orbit, the centripetal force mv^2/r on the orbiting body is the force of gravity GMm/r^2. This gives an orbital speed of $v = \sqrt{GM/r}$ and period $T = 2\pi r/v = 2\pi\sqrt{r^3/GM}$.

Bound and unbound orbits

If the total mechanical energy $K_{tr} + U_g$ of two bodies in their center-of-mass frame of reference is negative, they are gravitationally bound. If positive, they will escape.

The **escape speed** from a gravitational attractor of mass M depends on its mass and proximity: $v = \sqrt{2GM/r}$.

In a closed orbit, both bodies orbit the system's center of mass. Mechanical energy and angular momentum (about the center of mass) are both conserved.

Problems

1. The 1000-kg Lunar Reconnaissance Orbiter, in operation since 2009, orbits the Moon in a circular orbit 50.6 kilometers above the Moon's surface. Its orbital period is 1 hour, 54 minutes, and 34 seconds. The radius of the Moon is 1737.4 kilometers.
   a. What is the Moon's mass?
   b. What is the orbiter's total orbital energy (kinetic + potential)?
   c. What is the gravitational field at the Moon's surface?
2. The Moon orbits the Earth once every 27.3 days. The mass of the Earth is 5.976×10^{24} kg. What is the average orbital distance of the Moon?
3. Suppose two solar-mass (1.99×10^{30} kg) stars make up a double system with a constant separation between them of 1 AU (1.50×10^{11} m), the average distance between Earth and the Sun.
   a. What is their orbital speed?
   b. What is the period of their orbit?