Discussion 12: Sound

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In the interest of getting these solutions out before the exam, I have opted not to draw pictures this time around. I would still very much encourage drawing a picture to start every problem!

1. The sound level at a Formula 1 race is reported to reach 140 decibels at track-side, about 10 m from the cars. Long-term exposure to sound levels above 80 dB damages hearing; sound levels above 110 dB are uncomfortable, and levels above 130 dB are painful. Rodney Racefan has arrived at a Formula 1 race without his earplugs, and he doesn't want to pay for the expensive earplugs for sale at the track.

1a) How far from the track must Rodney stand to keep the noise he hears below the discomfort level of 110 dB?

To do this, we first need to find out how the intensities are related to eachother. We can do this using the given equation that relates decibels to intensity:

$$\beta_2 - \beta_1 = (10\ \text{dB}) \log_{10}\left(\frac{I_2}{I_1}\right), \tag{1}$$

Where β_2 is the decibel value of the sound at your initial position, and I_2 is that initial sound's intensity. Likewise, β_1 is the decibel value of the same source, with a different intensity, I_1. In our case, β_2 is 140 dB (the value at track-side, 10 m from the cars) and β_1 is 110 dB (the value at our distance r_1 from the car). Therefore, we have:

$$140 - 110 = (10\ \text{dB}) \log_{10}\left(\frac{I_2}{I_1}\right). \tag{2}$$

Simplifying a bit, I get:

$$3 = \log_{10}\left(\frac{I_2}{I_1}\right). \tag{3}$$

To undo my logarithm, I recall one of the logarithm rules:

$$\log_a(x) = b, \tag{4}$$

$$\Rightarrow a^b = x. \tag{5}$$

Therefore,

$$10^3 = \frac{I_2}{I_1}. \tag{6}$$

This tells us that:

$$I_2 = 1000 I_1. \tag{7}$$

This should make sense if we recall how we defined our subscripts. We defined β_2 as the dB value 10 meters from the cars, while β_1 was the dB value at some further distance. We would then expect I_2 to be a higher intensity than I_1, which is what we found. To figure out distances, we need to recall our definition of intensity:

$$I = \frac{P}{4\pi r^2}. \tag{8}$$

That means our two intensities are related to their distances by:

$$I_1 = \frac{P}{4\pi r_1^2}, \tag{9}$$

And

$$I_2 = \frac{P}{4\pi r_2^2}. \tag{10}$$

We can compare two intensities I_1 and I_2 by taking the ratio:

$$\frac{I_2}{I_1} = \frac{\frac{P}{4\pi r_2^2}}{\frac{P}{4\pi r_1^2}}. \tag{11}$$

We already know the left hand side. We found before that $\frac{I_2}{I_1} = 1000$. Therefore,

$$1000 = \frac{\frac{P}{4\pi r_2^2}}{\frac{P}{4\pi r_1^2}}. \tag{12}$$

Using our division rules:

$$1000 = \frac{r_1^2}{r_2^2}. \tag{13}$$

Bringing r_2^2 to the other side:

$$r_1^2 = 1000 r_2^2. \tag{14}$$

If we remember, we defined r_2 to be where the intensity was equal to I_2. That intensity had a decibel value of 140 dB. The problem told us that this was at $r_2 = 10$ meters. Therefore,

$$r_1^2 = (10)^2 \cdot 1000 = 100,000. \tag{15}$$

Taking the square root,

$$r_1 = \sqrt{100,000} = 316.23 \text{ m}. \tag{16}$$

This means that we have to stand 316.23 meters away from an object for a decibel difference of 30 dB.

1b) How far from the track must he stand to keep the noise he hears below 80 dB?

We can follow the same process as before. We start with:

$$\beta_2 - \beta_1 = (10 \text{ dB}) \log_{10}\left(\frac{I_2}{I_1}\right). \tag{17}$$

Our β_2 stays the same ($\beta_2 = 140$ dB). This time though, our $\beta_1 = 80$ dB. Therefore,

$$140 - 80 = (10 \text{ dB}) \log_{10}\left(\frac{I_2}{I_1}\right). \tag{18}$$

Simplifying, I get:

$$6 = \log_{10}\left(\frac{I_2}{I_1}\right). \tag{19}$$

Undoing the logarithm in the same way as before,

$$10^6 = \frac{I_2}{I_1}. \tag{20}$$

This tells us that I_2 is *a million* times more intense than I_1. We can use the same equation as before to compare these distances (see the previous part to follow how I got this):

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}. \tag{21}$$

Bringing the r_2^2 over and taking the square root:

$$r_1 = \sqrt{r_2^2 \frac{I_2}{I_1}} = r_2 \sqrt{\frac{I_2}{I_1}}. \tag{22}$$

Just as before, $r_2 = 10$ meters and we found $\frac{I_2}{I_1} = 10^6$. Therefore,

$$r_1 = 10\sqrt{10^6} = 10,000 \text{ m}. \tag{23}$$

Remembering that there are 1000 m in a km,

$$r_1 = 10 \text{ km}. \tag{24}$$

This means that we need to stand a whole 10 *kilometers* away from our source if we want to experience less than 80 dB of sound!

2. A spectator at a Formula 1 race (with proper hearing protection) notices that the sound from an approaching car has a frequency of 380 Hz, while the sound from the same car driving away has a frequency of 240 Hz. (The sound from a car is a combination of many frequencies and phases, but 380 and 240 Hz are the intensity maxima.) The speed of sound in air is 342 m/s.

2a) What is the speed of the car? Assume that it has the same speed when it is driving toward the spectator as when it is driving away.

To find the speed of of the car, we might consider using the Doppler Shift equation:

$$f_D = f_S \frac{v - v_D}{v - v_S}, \tag{25}$$

Where f_D is the frequency observed by the detected, f_S is the actual source frequency, v is the speed of sound (which we are taking to be $v = 342$ m/s), and v_S is the speed of the source. In our case, we are going to take our detector to be at rest, so $v_D = 0$. Therefore, what we actually have is:

$$f_D = f_S \frac{v}{v - v_S}. \tag{26}$$

The only problem here is that we have *two* unknowns. We don't know the actual source frequency, f_S *or* the speed of the source, v_S. When we have 2 unknowns, we need two equations to solve for them. Luckily, we *do* have to equations. We are told that the car's frequency when it is approaching is $f_{app} = 380$ Hz, and that the car's frequency when it is moving away is $f_{away} = 240$ Hz. Using our Doppler Shift equation, we can then say:

$$f_{app} = f_S \frac{v}{v - v_{app}}, \tag{27}$$

And

$$f_{away} = f_S \frac{v}{v - v_{away}}. \tag{28}$$

It might look like we have actually made the problem worse. We now have two equations and *three* unknowns. But we actually only have two. We are able to relate v_{app} and v_{away} if we read the problem carefully. The problem tells us that the car is moving at the same speed when it approaches as when it moves away. That means that the magnitudes of v_{app} and v_{away} are the same. The only thing that changes is our sign. Our equation sheet tells us that the convention is that a positive velocity is one that is moving *toward* the detector, and a negative velocity is one that is moving *away* from the detector. If our velocities have the same magnitudes, then that means that:

$$v_{away} = -v_{app}. \tag{29}$$

If we put this into one of our equations (either one works; here I will use the second, but you can just as well use the first) we then get:

$$f_{app} = f_S \frac{v}{v - v_{app}}, \tag{30}$$

And

$$f_{away} = f_S \frac{v}{v + v_{app}}. \tag{31}$$

Now we have two equations and two unknown. We can solve this using substitution. If we solve either equation for f_S (again, we could do either one; I will do the second), then we see:

$$f_S = f_{away} \frac{v + v_{app}}{v}. \tag{32}$$

We can put this equation for f_S into our other equation to get:

$$f_{app} = f_{away} \frac{v + v_{app}}{v} \frac{v}{v - v_{app}}. \tag{33}$$

Cancelling out the common v in the numerator and denominator:

$$f_{app} = f_{away} \frac{v + v_{app}}{v - v_{app}}. \tag{34}$$

Since our ultimate goal is to solve for v_{app}, we can then divide by f_{away}:

$$\frac{f_{app}}{f_{away}} = \frac{v + v_{app}}{v - v_{app}}. \tag{35}$$

Now we can multiply both sides by $v - v_{app}$ to get:

$$\frac{f_{app}}{f_{away}}(v - v_{app}) = v + v_{app}. \tag{36}$$

If we distribute out the left hand side:

$$\frac{f_{app}}{f_{away}} v - \frac{f_{app}}{f_{away}} v_{app} = v + v_{app}. \tag{37}$$

Then we can do some rearranging to get all v_{app} terms on the same side:

$$\frac{f_{app}}{f_{away}} v - v = v_{app} + \frac{f_{app}}{f_{away}} v_{app} \tag{38}$$

Pulling out v_{app} on the right hand side and v on the left hand side:

$$v\left(\frac{f_{app}}{f_{away}} - 1\right) = v_{app}\left(\frac{f_{app}}{f_{away}} + 1\right) \tag{39}$$

Dividing both sides by $\frac{f_{app}}{f_{away}} + 1$,

$$v_{app} = v \frac{\frac{f_{app}}{f_{away}} - 1}{\frac{f_{app}}{f_{away}} + 1}. \tag{40}$$

Although it's a bit ugly, this equation will help us find the velocity of our car. From here, we can put in numbers:

$$v_{app} = (342)\frac{\frac{(380)}{(240)} - 1}{\frac{(380)}{(240)} + 1} = 77.23 \text{ m/s}. \tag{41}$$

Therefore, the car is approaching our detector with a velocity of $v_{app} = 77.23$ m/s and moving away from our detector with a velocity of $v_{away} = -77.23$ m/s.

2b) What is the frequency of the sound emitted by the car?

Now that we know our car's speed, we can use either the approaching or receding Doppler Shift equations to find the actual frequency, f_S, of the car. If we use the approaching one:

$$f_{app} = f_S \frac{v}{v - v_{app}}. \tag{42}$$

We can solve this for f_S by dividing both sides by $\frac{v - v_{app}}{v}$:

$$f_S = f_{app} \frac{v - v_{app}}{v}. \tag{43}$$

Now we can put in numbers:

$$f_S = (380)\frac{(342) - (77.23)}{(342)} = 294.19 \text{ Hz}. \tag{44}$$

3. Let's look at the energy involved in heating water from room temperature (about 25 °C) to bath water temperature (about 40 °C).

3a) How much heat, in calories, must be added to 1.000 kilogram of 25 °C water to bring it to bath water temperature of 40 °C?

We know that it take 1 calorie to raise 1 gram of water by 1 degree Celsius. If we instead want to raise 1000 grams (or 1 kg) of water by 1 degree, we will need 1000 times as much energy, or 1000 calories. If we want to raise 1 kg or water by 15 degrees (from room temp to bath water temp), we will need 15 times as much energy, or 15000 calories.

3b) How much energy is that in Joules?

Here we need to convert our calories to Joules. We know that there are 4.184 Joules in a calorie. Therefore,

$$15000 \text{ cal}\frac{4.184 \text{ J}}{1 \text{ cal}} = 62,670 \text{ J}. \tag{45}$$

3c) If that amount of work is done to push 1.000 kilogram of water initially at rest, what is the water's final kinetic energy?

Here, we can use the Work-Energy Theorem, that states that the work done on a system is equal to its change in kinetic energy:

$$W = \Delta K. \tag{46}$$

If our water is initially at rest, then $\Delta K = K_f - K_i = K_f$. Therefore,

$$W = K_f. \tag{47}$$

If our work is 62,670 J, then:

$$K_f = 62,760 \text{ J}. \tag{48}$$

3d) What is the water's final speed?

Here, we use the definition of kinetic energy:

$$K = \frac{1}{2}mv^2. \tag{49}$$

Solving for v:

$$v = \sqrt{\frac{2K}{m}}. \tag{50}$$

For the kinetic energy found before, and 1 kg of water:

$$v = \sqrt{\frac{2(62,760)}{1}} = 354 \text{ m/s}, \tag{51}$$

Which is faster than the speed of sound!

3e) If that amount of work is done to lift 1.000 kilogram of water vertically, to what height is it lifted?

Another equation that relates the work done on an object to its energy is:

$$W = -\Delta U, \tag{52}$$

Where U is the potential energy. Writing it out, we have:

$$W = -(U_f - U_i). \tag{53}$$

Here, the negative sign comes from our definition of work:

$$W = \vec{F} \cdot \vec{r}. \tag{54}$$

If $\vec{F}$ is directly opposite $\vec{r}$, then our work is negative (we are working *against* the force). In the case of gravity, a positive vertical displacement will lead to negative work done on the system. Therefore, our 62,760 J of work is *actually* -62,760 J. If we take our initial height to be zero (making $U_i = 0$), then:

$$-62,760 = -U_f. \tag{55}$$

We can get rid of the negatives and put in the definition of U_f:

$$62760 = mgh. \tag{56}$$

Solving for h:

$$h = \frac{62760}{mg}. \tag{57}$$

For 1 kg of water on Earth ($g = 9.8$):

$$h = \frac{62760}{(1)(9.8)} = 6404 \text{ m}. \tag{58}$$

This is a *big* change in height! Especially when we consider that this is only the amount of energy it takes to raise the temperature of 1 kg of water by 15 degrees.