Discussion 13: Engines and Entropy

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1. A system receives 100 J of heat and does 125 J of work on surroundings.

1a) Is this possible?

The first law of thermodynamics states:

$$\Delta U = Q - W, \tag{1}$$

Where ΔU is the change in internal energy of a system, Q is the energy added to the system by heat, and W is the energy lost from the system by doing work. If the system receives 100 J of heat, then $Q = 100$ J, and $W = 125$ J is the energy lost from the system due to work. According to the first law:

$$\Delta U = 100 - 125 = -25 \text{ J}. \tag{2}$$

This is where we might encounter some discomfort. Is $\Delta U = -25$ J *really* physically possible? It is important to keep in mind that all this equation tells us is the *change* in energy for the system. It does not tell us anything about our initial or final conditions. A negative ΔU tells us that the internal energy of the system *decreases* by 25 J. It is entirely possible that the internal energy of a system could decrease by that amount, as long as U_f and U_i do not end up negative.

1b) Give a reason for your answer?

It is entirely possible for a system to do work at the expense of some of its internal energy. An adiabatic expansion (increasing volume with no heat exchange) will do work on its surroundings, possibly causing the system to cool down (which is a decrease in internal energy).

2. For each of the following irreversible processes, explain how you can tell that the total entropy of the universe has increased. Has matter spread out? Has energy spread out?

2a) Stirring salt into a pot of soup.

First let's convince ourselves that this is, in fact, an irreversible process. For the process to be truly reversible, we would have to be able to get back to our original state after we make changes. When we dissolve salt into soup, there isn't any way we are able to reverse the process and get back to separate salt and soup components.

As far as entropy goes, we are spreading out matter (in the form of salt), which will lead to the total entropy of the universe increasing.

2b) Scrambling an egg.

Scrambling an egg is intuitively an irreversible process. There is no way that we are able to separate the yolk and white of the egg after we have scrambled them.

The word *scrambled* is kind of a dead giveaway here. We are spreading matter out, which will lead to an increase in entropy. The white and yolk are no longer constrained to their original positions.

2c) A wave hitting a sand castle.

When a wave hits a sand castle, the wave is going to take some of the sand with it. In other words, the matter that made up the sand castle is getting spread out. The sand is no longer limited to the original positions in the castle. This will lead to an increase in entropy.

2d) Cutting down a tree.

This is another example of an intuitively irreversible process. When you cut down a tree, you cannot put it back together again.

The key thing here is that energy is spreading out. When the tree falls, its gravitational potential energy will turn into kinetic energy. As it impacts the ground, this kinetic energy will again change into other forms of energy (the impact will heat the system up, cause sound, etc.).

2e) Burning gasoline in an automobile.

In this case, both matter and energy are being spread out. When our liquid gasoline becomes a gas, the potential energy of the liquid becomes thermal energy of the carbon dioxide and water that is produced. The chemical equation that can be used to describe this is:

$$2C_8H_{18} + 25O_2 \rightarrow 16CO_2 + 18H_2O. \tag{3}$$

We can see here that our 2 moles of liquid (C_8H_{18}) and 25 moles of gas (O_2) becomes 34 moles of gas (18 moles of CO_2 gas and 16 moles of water vapor).

3. An ice cube (mass 30 g) sits on the kitchen table, where it gradually melts. The temperature in the kitchen is 25 °C.

3a) Calculate the heat transferred from the kitchen to the ice at 0 °C. The latent heat of fusion of water is $3.33 \cdot 10^5$ J/kg.

This is a phase change, so we can use the latent heat equation to find the heat transferred:

$$Q = mL, \tag{4}$$

Where it is given that $m = 30$ g (or $m = 0.03$ kg) and $L = 3.33 \cdot 10^5$ J/kg. Therefore,

$$Q = (0.03 \text{ kg})(3.33 \cdot 10^5 \text{ J/kg}) = 9990 \text{ J}. \tag{5}$$

3b) Calculate the change in the entropy of the ice cube as it melts into water at 0 °C. (Dont worry about the fact that its volume changes a little.)

Luckily, our ice stays at a constant 0 degrees, so we can use:

$$\Delta S = \frac{Q}{T}. \tag{6}$$

It is important to work in temperature units of Kelvin rather than degrees in thermodynamics. 0 degrees Celsius is equal to 273.15 Kelvin. Therefore,

$$\Delta S = \frac{9990 \text{ J}}{273.15 \text{ K}} = 36.57 \text{ J/K}. \tag{7}$$

3c) Calculate the change in the entropy of the kitchen as the ice cube melts.

The kitchen's temperature is given as 25 degrees Celsius. To convert from Celsius to Kelvin, we add 273.15. Therefore, the kitchen's temperature is 298.15 Kelvin. As far as the heat exchange goes, it is the same magnitude as the heat for the ice. However, the kitchen is *losing* this heat to the ice, so it is negative. Therefore,

$$\Delta S = \frac{-9990}{298.15} = -33.51 \text{ J/K}. \tag{8}$$

3d) Calculate the total entropy change of the universe to melt the ice.

Our last result might be a little concerning. The second law of thermodynamics tells us that the total entropy of an isolated system should always increase. However, we just found that the entropy of the kitchen is actually decreasing. Luckily, the kitchen isn't actually an isolated system. It is exhanging heat with the ice cube, so we will also need to consider its change in entropy to make a truly isolate system. We can also consider this to be the total change in entropy for the Universe:

$$\Delta S_{tot} = \Delta S_{ice} + \Delta S_{kitchen} = 36.57 - 33.51 = 3.06 \text{ J/K}. \tag{9}$$

Whew! We didn't break the second law of thermodynamics. The total change in entropy is an increase.

3e) Is melting the ice spontaneous?

In general, we consider a spontaneous event to be one that increases the total entropy of the Universe. As we just found, melting the ice *does* increase the total entropy of the Universe, so melting ice is a spontaneous event.

4. In places where it gets hot in the summer (unlike Laramie), it is common to install air conditioners to cool living spaces.

4a) Why must you put an air conditioner in the window of a building, rather than in the middle of a room?

Air conditioners generally take some amount of heat Q_c to do the work W that they need to do to cool a room down. In the process, they expel a certain amount of "waste" heat, Q_h, which is, in general, greater than the heat they extracted according to the following equation:

$$Q_h = W + Q_c. \tag{10}$$

Therefore, if an AC was in the center of a room, it would actually cause a net *increase* in the heat of the room. That is why we need our ACs to expel that heat outside and not inside.

4b) Can you cool off your kitchen by leaving the refrigerator door open? Explain.

This is the same idea as the AC. Just as with the AC, the refrigerator expels a certain amount of heat that is, in general, greater than the heat it takes to operate. To use a refrigerator to cool a room down, we would have to find some way to vent the excess heat outside.

4c) Estimate the maximum possible coefficient of performance of a household air conditioner. Use any reasonable values for the reservoir temperatures.

We might consider reasonable temperatures to be around $T_h = 40$ degrees Celsius = 313.15 K outside (a hot day) and $T_c = 25$ degrees Celsius = 298.15 K inside. We know that the coefficient of performance for a refrigerator is:

$$COP \leq \frac{T_c}{T_h - T_c}. \tag{11}$$

With our numbers:

$$COP \leq \frac{298.15}{313.15 - 298.5} = 19.9. \tag{12}$$

5. At a power plant that produces 1 GW (10^9 Watts) of electricity, the steam turbines take in steam at a temperature of 500 °C, and the waste heat is expelled into the atmosphere at 20 °C

5a) What is the maximum possible efficiency of this plant?

First, we should convert our temperatures to Kelvin. Our hot temperature is:

$$T_h = 500 + 273.15 = 773.15 \text{ K}. \tag{13}$$

And our cold temperature is:

$$T_c = 20 + 273.15 = 293.15 \text{ K}. \tag{14}$$

The efficiency is given by:

$$e \leq 1 - \frac{T_c}{T_h}. \tag{15}$$

With our temperatures,

$$e \leq 1 - \frac{293.15}{773.15} = 0.621, \tag{16}$$

Or an efficiency of 62.1%.

5b) How many kilowatt hours of electrical energy does this plant produce in a year?

The power is 1 gigawatt, so the total work is equal to 1 GW year. From here, we need to convert to kilowatts and hours:

$$\frac{1\text{GW yr}}{1}\frac{10^9 \text{ W}}{1 \text{ GW}}\frac{1 \text{ kW}}{10^3 \text{ W}}\frac{365 \text{ days}}{1 \text{ yr}}\frac{24 \text{ hr}}{1 \text{ day}} = 8.76 \cdot 10^9 \text{ kWh} \tag{17}$$

5c) Suppose you install pipes and turbines made from a new material that allows the maximum steam temperatures to be raised to 600 °C. What is the maximum efficiency of a plant that takes in steam at 600 °C and expels waste heat to the atmosphere at 20 °C?

Our hot temperature becomes:

$$T_h = 600 + 273.15 = 873.15. \tag{18}$$

The efficiency is then:

$$e \leq 1 - \frac{293.15}{873.15} = 0.664, \tag{19}$$

Or 66.4%.

5d) How many kilowatt hours of electrical energy will the plant produce in a year from the same fuel consumption (heat input Q_h) as before?

We know that in general,

$$W = eQ_h, \tag{20}$$

Or the work that a system is able to do is equal to the heat consumption multiplied by the efficiency. In the original case:

$$W_1 = e_1 Q_{h,1}, \tag{21}$$

And in the new case:

$$W_2 = e_2 Q_{h,2}. \tag{22}$$

However, we are able to assume that the heat consumption is the same in both problems ($Q_{h,1} = Q_{h,2} = Q_h$), so what we really have is:

$$W_1 = e_1 Q_h, \tag{23}$$

And

$$W_2 = e_2 Q_h. \tag{24}$$

This is a system of two equations with two unknowns (W_2 and Q_h). Therefore, we can solve for Q_h in terms of W_1 to get a solution for W_2. Our equations tell us that:

$$Q_h = \frac{W_1}{e_1}. \tag{25}$$

Putting this into our second equation:

$$W_2 = e_2 \frac{W_1}{e_1} = W_1 \frac{e_2}{e_1}. \tag{26}$$

Now we can put in our numbers:

$$W_2 = (8.76 \cdot 10^9 \text{ kWh}) \frac{0.664}{0.621} = 9.373 \cdot 10^9 \text{ kWh}. \tag{27}$$

5e) How much money will you make in a year by selling the additional electricity for 5 cents per kilowatt hour?

The total change in power is:

$$\Delta W = W_2 - W_1 = 9.373 \cdot 10^9 - 8.76 \cdot 10^9 = 6.13 \cdot 10^8 \text{ kWh}. \tag{28}$$

If I sell this excess at 5 cents per kWh, then:

$$6.13 \cdot 10^8 \frac{0.05 \text{ dollars}}{\text{kWh}} = 30.6 \cdot 10^6 \text{ dollars}, \tag{29}$$

Or 30.6 *million* dollars!