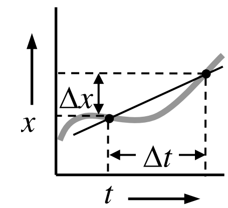
Discussion sheet 1: Straight-line constant acceleration kinematics

Summary

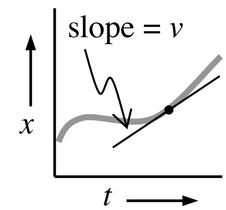


Average velocity is the ratio of how far an object moves to the time elapsed.

$$v_{avg} = \Delta x/\Delta t$$

On a plot of position vs. time, v_{avg} is the slope of the secant line connecting the starting and ending events.

Instantaneous velocity is the limit of average velocity as the time interval becomes infinitesimally brief. It is the velocity at a particular instant of time.



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

On a position-time plot, v is the slope of the tangent line at that particular instant. Conversely, the area under a velocity-time plot is the change in position.

Average acceleration is the rate of the change of an object's velocity.

$$a_{avg} = \Delta x/\Delta t$$

On a plot of velocity vs. time, a_{avg} is the slope of the line connecting the starting and ending events.

Instantaneous acceleration is the limit of average acceleration as the time interval becomes infinitesimally brief. It is the acceleration at a particular instant of time.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

On a of velocity-time plot, a is the slope of the tangent line at that particular instant. Conversely, the area under an acceleration-time plot is the change in velocity.

Kinematic Formulas

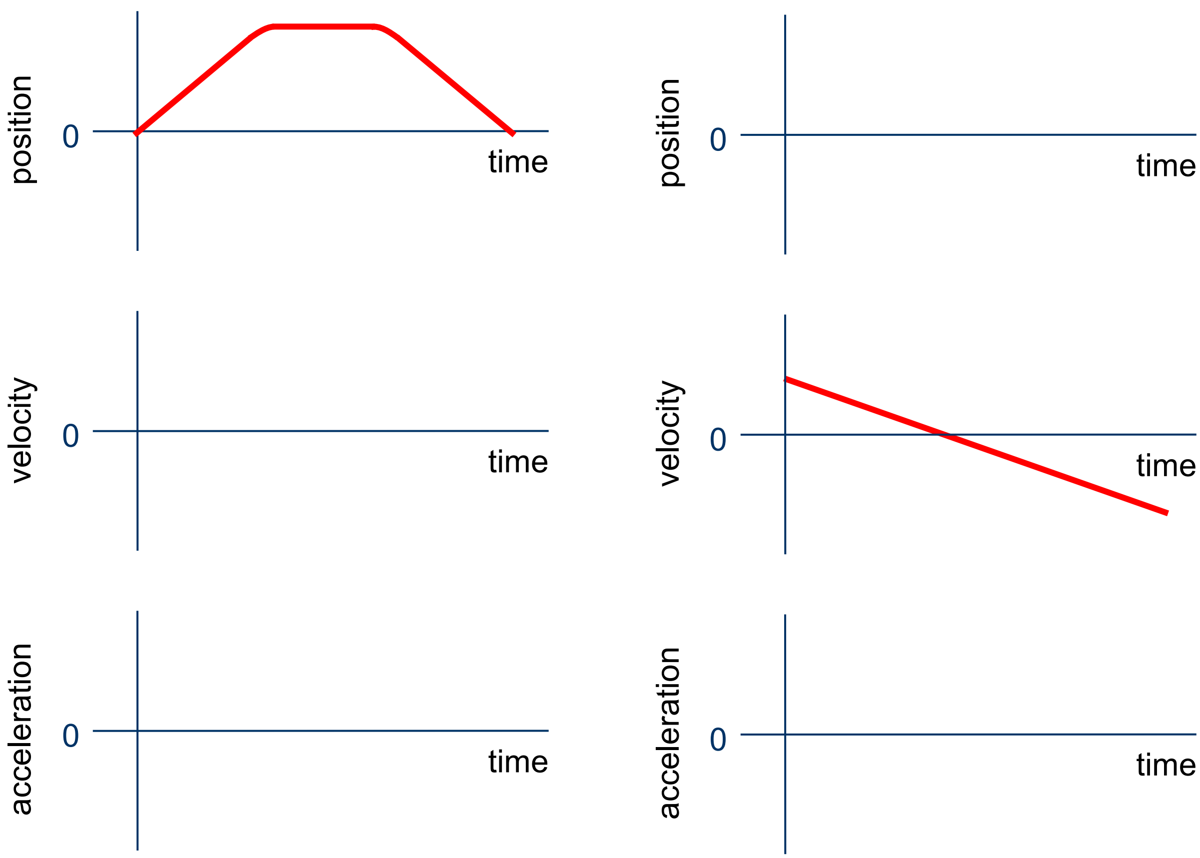
When velocity is constant, $\Delta x/\Delta t$ is the same for any time interval. Then $x = x_0 + vt$.

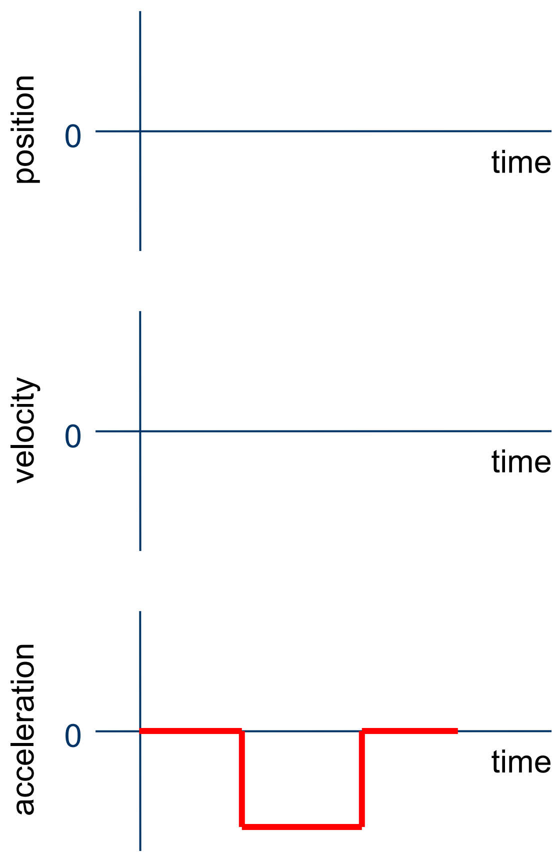
When acceleration is constant, $v = v_0 + at$; $x = x_0 + v_0 t + \frac{1}{2}at^2$.

Algebraic substitution allows us to find relations not requiring t or not requiring a:

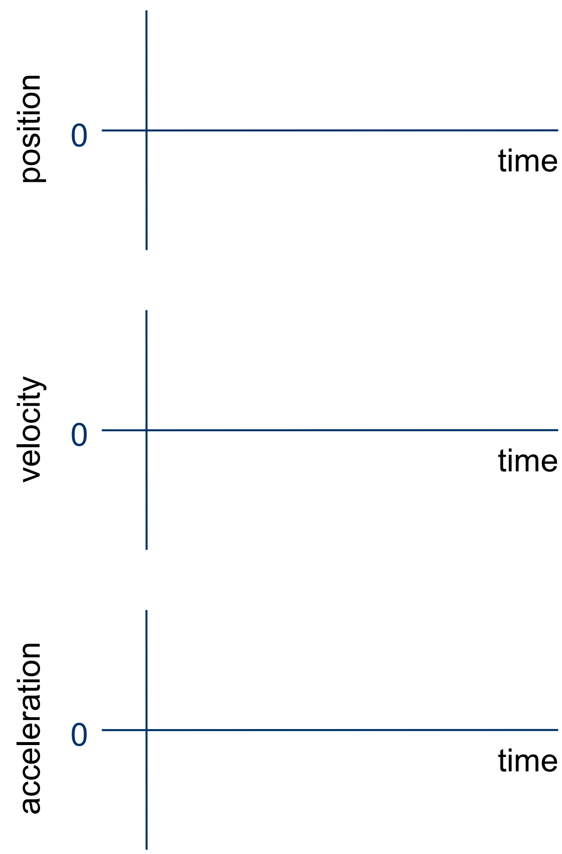
$$2a(x - x_0) = v^2 - v_0^2; \qquad x - x_0 = \frac{1}{2}(v_0 + v)t$$

1. In the following scenarios, the motion of an object is to be described in four ways: (i) in words, (ii) as a position-time graph, (iii) as a velocity-time graph, and (iv) as an acceleration-time graph. In each case, only one description is given. Construct the other three. (You may need to assume some initial conditions.) For additional fun, think of mathematical expressions that would describe the position, velocity, and acceleration.





A coconut hangs motionless from its tree, then drops with increasing downward speed until it lands on the ground, quickly coming to rest.



2. A ball starts from rest and rolls down an incline at a constant acceleration. In 5.0 s, it rolls a distance of 50.0 m down the hill.
   a. What is its acceleration?

   b. If the same ball rolls down the same incline with the same acceleration, but begins with an initial speed of 2.0 m/s downhill, how far down the hill will it be in 5.0 s?

   c. If the ball begins with an initial speed of 2.0 m/s *uphill*, where will it be in 5.0 s?

3. An automobile accelerates constantly from rest, traveling 400 m in 20.0 s.
   a. What is its average velocity over this interval?

   b. What is its final velocity?

   c. The car's engine is adjusted, and the car is again accelerated constantly from rest through a distance of 400 m. This time, its final velocity is 50 m/s. What is the time elapsed?

4. The human body can survive an acceleration trauma incident (sudden stop) if the magnitude of the acceleration is less than 250 m/s^2. If you are in an automobile accident with an initial speed of 30 m/s and you are stopped by an airbag that inflates from the dashboard, over what distance must you stop for you to survive the crash?

5. Serena lobs a tennis ball straight upwards. It rises a distance H before stopping and falling back down. Disregard air resistance.
   a. What is the initial speed v_0 of the tennis ball? Express in terms of height H and g, the acceleration due to gravity.

   b. How much time does it take for the ball to reach the top of its trajectory? Express in terms of H and g.

   c. When the first ball is exactly at the top of its trajectory, Serena lobs a second tennis ball straight up with the same initial speed v_0.
      i. How much time later are the balls at the same height? Express in terms of H and g.

      ii. What is the height at which the balls cross? Express in terms of H and g.