Lab 1. MEASUREMENT AND UNCERTAINTY

1.1 Problem

- Are measurements and analysis of errors fundamental to science?
- How do measurement errors affect the interpretation of scientific observations?
- How are measurements and error analysis used in laboratory experiments?

1.2 Equipment

Station 1: metal ruler, balance, micrometer, Vernier caliper, graduated cylinder

Station 2: ruler

Station 3: metal rod, steel ruler, Vernier calipers, graduated cylinder, water

Station 4: vacuum bottle filled with liquid nitrogen (or crushed dry ice if liquid nitrogen is not available), foam cups, safety goggles, thermal gloves, timer, balance

1.3 Background

Science describes the physical world through measurement. Almost all scientific experiments involve **measurement**, the assignment of numbers to certain properties. A fundamental assumption of science is that some of these properties are meaningful, and that the numbers associated with them are meaningful as well.

<u>Measurement Errors</u>

A fundamental limitation of measurement (and science) is that the number assigned to a quantity by measuring it is never exactly equal to the "true" value. This difference between a quantity's actual and measured values is known as **error**. Errors can be classified into two types based on how they originate.

Systematic errors are created by the experimental process itself and are consistent throughout an experiment. They tend to cause measurement errors to have some sort of pattern to them. For example, an incorrectly calibrated pump may consistently charge customers for more gasoline than is actually dispensed. The measurement errors in this case would all have the same sign, because the measured volumes would be greater than the true volumes. The larger the volume dispensed, the greater the measurement error would be.

Random errors result from factors not controlled by the experimental process. These factors will be unpredictably different for each measurement, creating measurement errors with no pattern. In the case of the gas pump, the meter may stop running shortly before or shortly after the pump shuts off. If the time difference varies from run to run, it adds an error that is different for each measurement.

The distinction between systematic and random errors can be ambiguous. Continuing with the gas pump example, if the fuel is at a significantly lower or higher temperature than the temperature at which the meter was calibrated, the mass of fuel in a given metered volume will be higher or lower than calculated. Whether this is considered a systematic or random error depends on the time span of the measurements and on the purpose of the experiment. It is more important for the experimenter to *understand* the error and how it affects the experiment's conclusions than to merely classify it as systematic or random.

Accuracy and Precision

Related to but distinct from error types is the precision and accuracy of measurement. **Accuracy** is a measure of similarity between a measurement and a standard value, which is correct by definition. In contrast, **precision** is a measure of similarity among individual measurements in a series of measurements. If repeated measurements of the same quantity always assign very close numbers, the measurement is said to be precise, even if the average is incorrect. If the measurements average near the correct number the measurements are said to be accurate. For example, suppose we are trying to cut table legs to a length of one foot, using a 12-inch ruler. Measuring a single leg multiple times, we find that our measurements give 12.0 inches, agreeing to within 0.1 inch. Therefore, we can say that these measurements are precise to 0.1 inch. However, suppose the ruler is warped, so that 12 inches on our ruler really corresponds to 11.75 inches. This means that the set of measurements has an accuracy of only 0.25 inch.

The amount of uncertainty is commonly implied by the number of "significant figures" in the number assigned during the measurement. Thus, a length measurement of 1.000 meters is implied to be more precise than a length given as 1.00 meters. If, for example, we were measuring lengths with a meter stick having a scale marked only with hundredths of a meter, it would not make sense to assign a length of 1.0000 meters. A fourth zero is meaningless since we cannot measure precisely enough to distinguish between 1.0000 and 1.0001 meters. We can, however, distinguish between 1.000 and 1.001 meters (or perhaps between 1.000 and 1.005 meters) in length. We would therefore assign 1.000 meters to our measurement.

The *accuracy* of a measurement can only be evaluated by comparing many measured values to their corresponding standard values. A measure of accuracy is the **percent absolute uncertainty**, %*A*, given by:

$$\%A = \frac{M - M_S}{M_S} \times 100\% \qquad (1)$$

where M is a specific measurement one has made and M_s is the standard value. The larger %*A* is, the less accurate the measurement. To determine %*A*, it is necessary to know the standard "true" value of the quantity being measured. This is seldom known except when calibrating the measuring process with a standard quantity.

The *precision* of a measurement can be evaluated by comparing many repeat measurements of the same quantity with each other. A measure of precision is the **percent relative uncertainty**, %*P*, given by:

$$\%P = \frac{M - \bar{M}}{\bar{M}} \times 100\% \qquad (2)$$

Where $\bar{M}$ M_a is the average value of a series of measurements. The larger %*P* is, the less precise the measurement. To determine %*P*, it is necessary to make a series of repeat measurements. The more measurements there are in the series, the more likely their average is close to the "true" average.

The precision of a measurement can be improved by combining repeat measurements. If you were to measure the length of a candle flame in a drafty room (hint, hint…), some of your measurements would be too large, and some too small. These random errors, introduced by the flickering of the flame, can be removed from the data by taking an average of a series of measurements. During the averaging process, the measurements that are too small will cancel those that are too large. As more and more measurements are made, the average value converges to the true value. Note that averaging will *not*, in general, cancel systematic errors.

1.4 Measuring Instruments

Here is an overview of the measuring instruments you will be using in this lab:

RULER: Simple as it is, the ruler or meter stick is often used incorrectly. Your eye must be positioned directly over the ruler when taking a reading, so that the ruler is perpendicular to your line of sight. If you try to read the ruler at an angle, your measurement will be too high or too low. (This is a common error called **parallax**.)

VERNIER CALIPERS: Many metric scales have one millimeter as their smallest division. You need to estimate the tenths of a division for a best measurement. A Vernier allows you to estimate this fractional part precisely. It has a main scale and along side of this, a Vernier scale. The Vernier scale has 10 marks in the length of 9 marks on the main scale. The left-most mark on the Vernier scale is the zero marker. Use it to read the whole number of divisions. Then obtain the fractional part of one division by observing which of the ten Vernier marks lines up best with a mark on the main scale. The number of this mark is the fractional digit of the measurement.

MICROMETER: A micrometer is another device used to measure length. A micrometer can directly measure lengths to 0.01 mm and can be used to estimate lengths to 0.001 mm. The key components of a micrometer are the anvil (against which the object to be measured sets), a moveable spindle, barrel, thimble, and ratchet. The spindle can be opened or closed using the ratchet mechanism. The dimension of the object being measured is read from the scales on the barrel and thimble. Your instructor will demonstrate how to use and read a micrometer.

DIGITAL BALANCE: Check the level indicator, if there is one. It will be a bubble inside a circular viewing window. If the bubble is not centered within the circle, adjust the adjustable feet of the balance to center it. Check that the balance reports its measurement in the desired units. Zero the balance by pressing the "tare" button. Wait for the display to return to zero. Place the object to be weighed on the pan. Wait for the display to become steady before recording the measurement.

GRADUATED CYLINDER: Graduated cylinders, like rulers, are subject to parallax errors. To most accurately read a graduated cylinder, place it on a stable level surface and position your eye at the same level as the surface of the liquid. The surface of the liquid will be an upward-curving meniscus; read the level of the *bottom* (center) of the meniscus.

Taking measurements

Physical measurements are used to determine many physical quantities including time, length, mass, etc. The precision of the measurements depends on the type of instrument that is used, the technique used to calibrate it, and the person making the measurements.

The **least count** of a measurement device is the smallest subdivision or unit that is marked on the scale of the device. One can usually estimate a measurement to 1/10 of the least count. Some instruments, such as a Vernier calipers, have scales that will allow much more accurate estimates of 1/10 of the least count.

1.5 Stations

Go to the experiment station that has the least number of people waiting and follow the instructions given below for that station. Make your measurements to the best precision that your measuring device will allow. Continue moving to different stations until you have finished all the experiments.

Station 1: Measuring Instruments

List the least count and the precision to which one can estimate a measurement for the following instruments. Specify the units for each.

Instrument	Least Count	Estimate to
Steel Ruler	________________	________________
Vernier Caliper (SI scale)	________________	________________
Micrometer	________________	________________
Graduated Cylinder	________________	________________
Balance	________________	________________

Station 2: Reaction Time

The time it takes to respond to a stimulus is called **reaction time**. Human reaction time can be measured in a variety of ways. A simple method to determine human reaction time uses the principle of uniform acceleration of a falling object. In the seventeenth century Galileo showed that the distance an object falls at the Earth's surface is directly related to the square of its time falling, if the object started at rest. You will use this consistent behavior of motion on Earth to determine your reaction time.

Practice this procedure several times before beginning to take actual measurements. Try to work as accurately as possible.

1. Have your lab partner hold the top of a ruler so that it is straight up and down.

2. Straddle the bottom of the ruler with your index finger and thumb to prepare to grab it. As soon as your partner releases the ruler (without telling you in advance), pinch it as fast as you can. Do not move your hand up or down—that would invalidate the measurement.
3. Read off the distance the ruler has fallen.
4. Do this at least 5 times and record your results in the "Distance" column of Table 1.

From the distance the ruler falls you can determine your reaction time, using the relationship originally discovered by Galileo:

$$t = \sqrt{\frac{2d}{980\ \mathrm{cm}/s^2}} \qquad (3)$$

where d is the distance in centimeters and 980 $\mathrm{cm/s^2}$ is the acceleration due to gravity. Typical reaction times are between 0.2 and 0.3 seconds. Fill in the "Reaction Time" column of Table 1 with your results. (You can do this on a spreadsheet if you wish.)

Table 1. Reaction Time

Trial	Distance (cm)	Reaction Time (s)	$t - t_{avg}$ (s)	$\%P$
1				
2				
3				
4				
5				

Average reaction time t_{avg} = ____________ s.

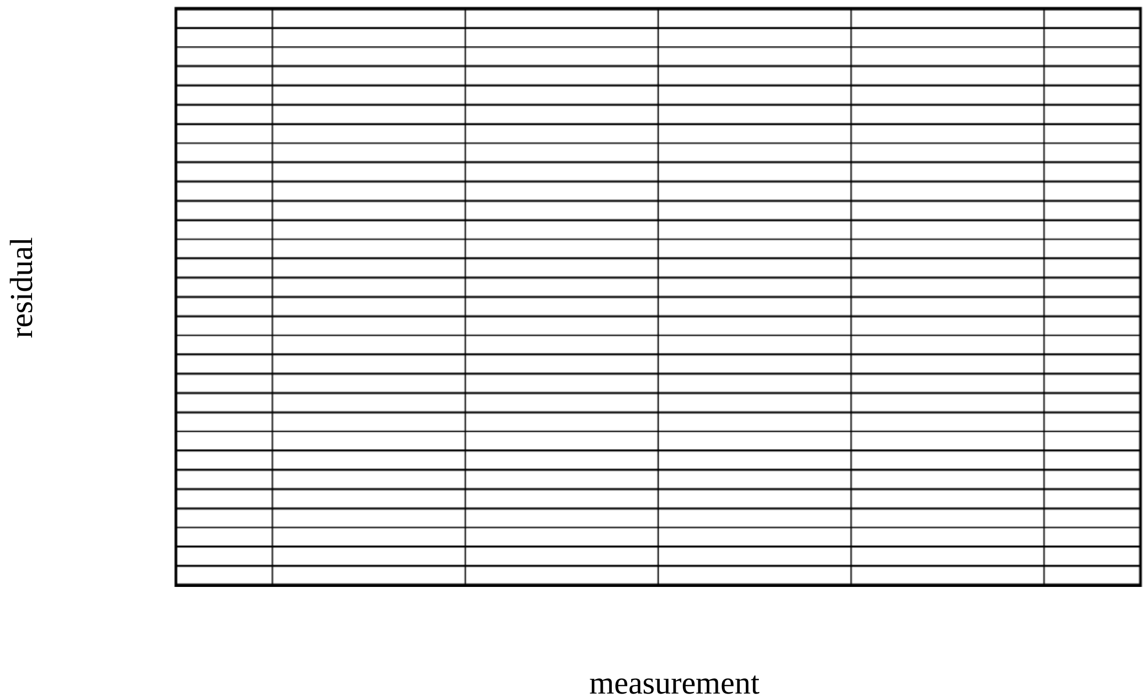
Calculate the average reaction time and enter it above. Use it to determine the "residuals" $t - t_{avg}$ and percent errors $\%P$. Enter these values into the table as well.

Questions

1. What steps did you take to be sure you measured the exact distance the ruler fell?

2. What steps did you take to prevent the catcher from anticipating a release?

3. Make a scatter plot of the residuals below. Scale the vertical axis to use at least half of the available space. Mark the scales on your axes and title your graph



Station 3: Mass, Volume, and Density

Mass and volume are *extensive* properties. This means that these values are proportional to the amount of substance present. Volume may be calculated for regularly shaped objects (cubes, right circular cylinders, etc.) by measuring the dimensions and applying the appropriate formula. For irregularly shaped objects, the volume may be determined by displacement of a liquid.

Density is an *intensive* property of a material: the value does *not* depend upon the amount of material present. Density can be calculated by dividing the mass of an object by its volume:

$$\text{density} = \text{mass}/\text{volume}.$$

Part 1: Measure the length of a metal rod to 1/100 cm using a steel ruler. Have four other people in the lab measure the same rod and record the values in the first column below.

Measurement #	Length of Rod x (cm)	$x - \bar{x}$	$(x - \bar{x})^2$
1	________	________	________
2	________	________	________
3	________	________	________
4	________	________	________
5	________	________	________
		$\sum(x_i - \bar{x})^2$	________

Calculate $\bar{x}$, the average value of the length of the rod. $\bar{x}$ = ________. Calculate the deviation from the mean and the square of the deviation from the mean for each measurement. Calculate the standard deviation (σ) for the data using the following formula. Don't forget the units on the standard deviation.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

σ = ______________.

Using the Vernier calipers, measure the length and diameter of the metal rod. Compare the Vernier caliper measurement with the *average* value of the *length* found by using the steel ruler. Calculate the % Difference between the two values using:

$$\%\text{Difference} = \frac{x_{\text{ruler}} - x_{\text{Vernier}}}{x_{\text{average}}} \times 100\%$$

where x_{average} is the average of x_{ruler} and x_{Vernier}. Calculate the volume of the metal rod (volume = $\pi r^2 l$) using the length obtained with the Vernier caliper.

Length ____________ Diameter ___________

Method	**Length (cm)**
Vernier Caliper (x_{Vernier})	__________
Steel Ruler Average Value (x_{ruler})	__________

% Difference _____________.

Volume of metal rod_______________.

Part 2: Using the balance, find the mass of the metal rod and compute its density based upon the volume calculated previously.

Mass of rod ______________

Computed volume of rod ______________

Density using computed volume ______________

Determine the volume of the rod by displacement of water in a graduated cylinder and compute its density. **Do not drop the rod** into the graduated cylinder: the cylinder would probably break. Instead, tip the cylinder so that it is nearly horizontal and slide the rod to the bottom of the cylinder. Then right the cylinder.

Calculate the density of the bar by using the formula mass/volume with the volume measured by water displacement.

Volume of rod by displacement ______________

Density using displacement ______________

Metal Density Table

Metal	**Density, kg/m^3**
Aluminum	2.7×10^3
Brass	8.6×10^3
Copper	8.9×10^3
Steel	7.8×10^3

From the Metal Density Table, identify the material from which the metal rod is made. Calculate the % Error of the calculated densities using

$$\%\ \text{Error} = \frac{x_{\text{measured}} - x_{\text{table}}}{x_{\text{table}}}$$

Identity of metal ________________

Density of metal from Metal Table ________________

% Error from calculated volume ________________

% Error from displacement volume ________________

Question

Which of the densities that you calculated in Part 2 is most accurate? Why?

Station 4: The Mass of Liquid Nitrogen (or) The Mass of Water and Dry Ice

NOTE: You only have to do one of the following procedures. **In either of these experiments, all group members must wear goggles, and the person handling the nitrogen or dry ice must wear gloves.**

Liquid Nitrogen:

1. Take a small foam cup and fill it with liquid nitrogen.
2. Wait for about one minute until the liquid nitrogen in the cup settles to a steady, slow boil.
3. Measure the mass of the cup of liquid nitrogen with a balance at 5 different times, 30 seconds apart. When handling liquid nitrogen, take extreme care to avoid any contact with your skin.
4. Record your measurements in Table 2.

Dry Ice:

1. Half fill a foam cup with crushed dry ice.
2. Fill the cup to the top with water.
3. Measure the mass of the mixture with a balance five separate times, 30 seconds apart. Skin burns could result from extensive contact with the dry ice. Do not touch the dry ice with bare hands. Also, sometimes the dry ice 'pops' out of the cup. This is why goggles are required.
4. Record your measurements in Table 2.

Table 2. Mass of ______________________

Trial	Mass (g)	$m - m_{avg}$ (g)	$\%P$
1			
2			
3			
4			
5			

Average mass m_{avg} = __________________ g.

Calculate the average mass and enter its value in the space above. Use it to determine the "residuals" $m - m_{avg}$ and percent errors $\%P$. Enter these values into the table as well.

Question

1. Are the differences between successive measurements due to systematic error or random error?

2. Make a scatter plot of the residuals below. Scale the vertical axis to use at least half of the available space. Label the scales and title your graph.