LAB 10. HOOKE'S LAW

Introduction

External force applied to an object will change the object's size or shape or both. Whether the object springs back to its original shape after the force is removed (elastic response) or remains deformed (plastic response) depends on the arrangement and bonding of the atoms in the material as well as the magnitude, rate and duration of force applied. The simplest approximation to the elastic behavior of an solid is Hooke's law, $F = -kx$, where F is the force exerted by the solid, k is the stiffness of the solid, and x is its distortion from its equilibrium size. Springs are constructed to display this behavior over fairly large extensions.

If a mass m is acted on only by a Hooke's law spring with force constant k, it oscillates with a period $T = 2\pi\sqrt{m/k}$.

In this lab you will use these two relations to obtain two different estimates of the stiffness k of several springs.

Theory

The extension x of a spring is its change in length: if its resting length is l_0 and its length under tension or compression is l, then $x = l - l_0$. Hooke's law $F = -kx$ tells the force F exerted by a spring as a function of the extension of the spring. In this lab, we will stretch a spring by hanging known masses from it. It will be natural for us to measure the static extension of the spring as a function of applied weight w, which by Newton's third law is $-F$. Then we can predict the length of the spring in response to the applied weight.

$$-w = F = -kx$$

$$w = kx = k\,(l - l_0) = kl - kl_0$$

$$kl = w + kl_0$$

$$l = w/k + l_0$$

Consequently, if Hooke's law is true, a plot of the length l of a spring vs. applied weight w should give a straight line with slope $1/k$ and y-intercept (actually l-intercept) l_0.

Likewise, we can measure the periods of oscillation T of different masses m on the spring. Rearranging the expression above for period gives

$$T^2 = 4\pi^2 m/k.$$

This tells us that a plot of T^2 vs. m should give a straight line through the origin with slope $4\pi^2/k$.

Equipment

Two labeled springs, clamp stand, clamp, masses, stopwatch, meter stick.

Activities

Static tension

1. Measure and record the identifying number and initial length l_0 of a spring.
2. Hang a known mass from the spring and allow the mass and spring to come to rest. Record the spring's length l_1.
3. Find a set of at least seven (7) different masses that you can hang from the spring. They should all be heavy enough that they stretch the spring beyond l_0 and light enough that they do not over-stretch it. Record the masses and the corresponding static spring until you have seven (7) measurements $l_1 \ldots l_7$ in addition to the initial length l_0. Record these additional data as well.
4. Convert the masses to the forces they exert on the spring by multiplying by the gravitational field g. In other words, $w = mg$. Record these forces.
5. Repeat the process with a different spring.

Oscillation

1. Record the identifying number of the spring.
2. Hang the spring from a rigid clamp or horizontal bar.
3. Find a set of at least five (5) different masses that you can hang from the spring. They should all be light enough that they do not over-stretch the spring, and heavy enough that they stretch the spring enough to clearly observe their oscillations. Record their masses.
4. Hang a mass from the spring. Raise and release the spring so that it oscillates. Ensure that it oscillates purely up and down; stop it and begin again if it swings from side to side. Also ensure that the mount remains fixed, and does not move with the oscillator.
5. Time a whole number of complete cycles of the oscillation. Use at least ten oscillations; more if the oscillation is rapid. Start the timer on "zero" and stop at the desired number of oscillations. You don't need to start timing when you release the weight: you can get more accurate measurements if you start timing at a later cycle.
6. Stop the oscillations and start them again. Time the same number of oscillations. Repeat for three runs. Record the times and the number of oscillations.
7. Repeat the procedure for all of the different masses, three runs for each mass.
8. Repeat the process with the other spring.

Data

Static tension

Spring ______________	
Hanging mass m	Length l
0	

Slope ___________________

Intercept ________________

Estimated k ______________

Spring ______________	
Hanging mass m	Length l
0	

Slope ___________________

Intercept ________________

Estimated k ______________

Oscillation

Mass m	Cycles N	Time NT		

Slope ___________________

Intercept ________________

Estimated k ______________

Mass m	Cycles N	Time NT		

Slope ___________________

Intercept ________________

Estimated k ______________

Data Processing

Tension and extension

For each spring:

1. Plot a scatter graph of length (vertical axis) vs. load (horizontal axis).
2. Fit the scatter plot with a linear trend line $y = ax + b$. Record the parameters a (slope) and b (intercept) above.
3. Calculate an estimate of the spring constant k from the slope parameter a.

Mass and Period

For each spring:

1. Divide the times by the number of oscillations to find an estimate of the period T for each run. Average these to find one T for each mass.
2. Make a scatter plot of T^2 (vertical axis) vs. m (horizontal axis).
3. Fit the scatter plot with a linear trend line, $y = ax + b$. If you are using Excel, in the "Add Chart Element" menu, choose "Trendline" and "Linear." Under "More Trendline Options," select "Display Equation on chart." Record the trend line parameters a (slope) and b (intercept) above.
4. Calculate the estimate of k from the slope parameter a.
5. Note the intercept parameter b. If the Hooke's law model applies, b should be close to zero.

Check-out

Data and graphs

Show your instructor your data and graphs.

Questions

Discuss these questions with your instructor.

- Does Hooke's law appear to adequately describe static tension data? Identify evidence supporting your answer.
- Do your observations or graphs provide evidence for or against the model that $T = 2\pi\sqrt{m/k}$?
- Compare the estimates of k from the static tension activity to the estimates from the oscillation activity. Do they agree? If they do agree, what does that mean? If they do not agree, what does that mean?