Lab 12. ENTROPY

Problem

- If individual objects behave randomly and independently of each other, do collections of many of them behave predictably?
- Why do matter and energy tend to disperse over time?

Equipment

Board divided into five zones, ten cups, 100 dice, cafeteria tray, tweezers, pencil, sheet of paper

Background

Situations in which many small objects, individually behaving randomly, combine to form a predictable whole occur frequently in physics. In this activity we will explore the basis of the physical property known as *entropy*, the tendency of matter and energy to spread out rather than to accumulate in a particular location.

Entropy

All matter consists of microscopic particles that follow the same physical laws that govern larger objects: they are accelerated by gravitational fields, any force they exert on another object is matched by an opposite force acting on them, and so on. We refer to these particles as molecules, although they could in fact be electrons, individual atoms, molecules in the chemical sense, or groups of molecules clumped together.

We will explore trends over time of two characteristics of molecules: location and kinetic energy. Location is where the molecule is; kinetic energy depends on the speed of the molecule, without regard to the direction it is moving. If every molecule in a large sample has a small chance of moving away from its position, with motion in all directions equally likely, how does the large sample itself move about over the course of time? Likewise, when two molecules interact, if every molecule has an equal chance of either transferring some of its kinetic energy to the other molecule or of taking some kinetic energy from the other molecule, how will kinetic energy become distributed among the molecules?

Entropy is a technical term that quantitatively describes how evenly distributed some physical quantity is: high entropy means that molecules are spread out to fill all available space, and kinetic energy is evenly distributed so that molecules tend not to have very much more or less than the average. A critical principle in physics is that in a closed system, that is, one that neither receives energy from nor loses energy to its surroundings, *entropy increases as time progresses*. A state of maximum entropy, in which no further increase is possible, is *equilibrium*. Sometimes this is achieved by spreading out molecules, and sometimes by spreading out energy. Sometimes both matter and energy become more spread out. Sometimes, however, one spreads out while the other becomes more localized. For that to occur, the system must gain more entropy from spreading out the one than it loses from localizing the other.

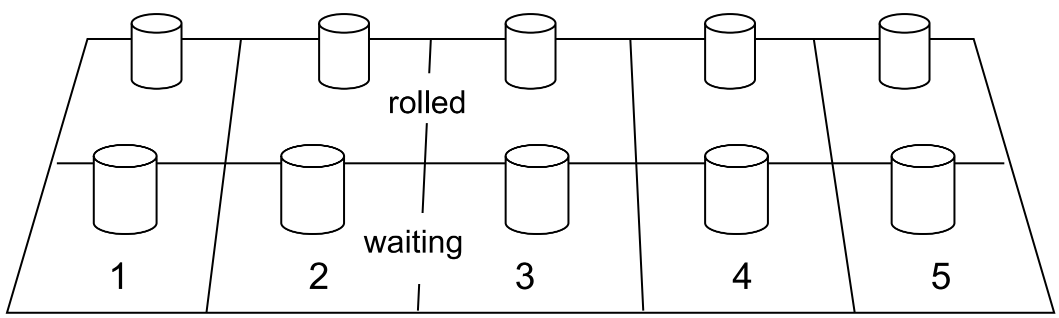
Activity

You will investigate how individual molecules, with no preference for moving to the left or to the right, move among the five zones of a game board. In a given step, a molecule may either move one space to the left, stay where it is, or move one space to the right. The molecules behave completely independently of each other; whether a molecule moves and which direction it moves are not affected by where any of the other molecules are or how they move.

Each molecule is represented by a six-sided die, and its movement is determined by the roll of the die. A roll of 1 means that the molecule moves one zone to the left, a roll of 2–5 means that the molecule stays in place, and a roll of 6 means that the molecule moves one zone to the right. If a molecule is already in the far-left zone (1), a roll of 1 is ignored; correspondingly, if the molecule is in the far-right zone (5), a roll of 6 has no effect.

Procedure:

1. Place one cup in each of the ten regions of the poster board: “rolled” and “waiting” of each of zones 1–5.

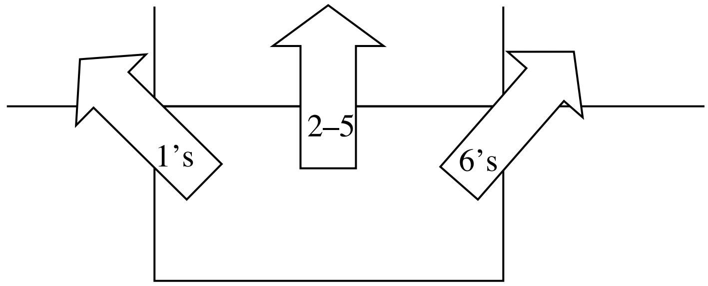


2. Verify one hundred (100) dice by arranging them into a 10 × 10 square.
3. Put all of the dice into the “waiting” cup of Zone 3.
4. In the Table, Step 0, record that all 100 dice are in Zone 3.

First Roll:

5. *Gently* roll the dice into the tray. The tray keeps them from rolling off the table, getting lost, or becoming mixed with another group’s dice.
6. Remove all dice that come up “1” and put them into the “rolled” cup of the zone to the left. (That is Zone 2.)
7. Remove all dice that come up “6” and put them into the “rolled” cup of the zone to the right. (That is Zone 4.)

8. Remove all the remaining dice (rolled "2" through "5") and put them into the "rolled" cup of Zone 3. These are the molecules that do not move in this step.



9. Record the number of dice in each zone at the conclusion of this step (1) in the Table.
10. All the dice should be in the "rolled" cups in their zones. Switch each "rolled" cup with the "waiting" cup in the same zone so that all dice are now in the "waiting" cups.

Second and Later Rolls:

11. If there are any dice in the "waiting" cup for Zone 1, roll them. Place the rolled dice into the "rolled" cup of the zone where there numbers direct them to go: 1's–5's remain in Zone 1, and 6's go to Zone 2.
12. Now, if there are any dice in the "waiting" cup for Zone 2, roll them. Place dice coming up "1" into the Zone 1 "rolled" cup, dice coming up "2", "3", "4", or "5" into the Zone 2 "rolled" cup, and dice coming up "6" into the Zone 3 "rolled" cup.
13. Carry out something like step 12 for Zones 3 and 4. Roll only the dice in the "waiting" cups, not in the "rolled" cups! Dice coming up "1" go into the "rolled" cup of the zone to the left, dice coming up "6" go into the "rolled" cup of the zone to the right, and all other dice go into the "rolled" cup of the zone where they were.
14. Finally, if there are any dice in the "waiting" cup for Zone 5, roll them. Dice coming up "1" go into the "rolled" cup of Zone 4; and dice coming up "2"–"6" go to the "rolled" cup of Zone 5.
15. Count the number of dice in each zone and record in the Table. Switch the "rolled" cup for the "waiting" cup in each zone.
16. Repeat steps 11–15 until you have completed all 20 rows of the Table.
17. Ω is the "multiplicity" of that arrangement of dice: the number of different ways the 100 dice can be placed in the zones while maintaining n_1 dice in zone 1, n_2 dice in zone 2, and so on. The full formula is (I think) $\Omega = 100!/(n_1!\,n_2!\,n_3!\,n_4!\,n_5!)$. Here $N!$ means "N factorial:" $N! = N \cdot (N-1) \cdot (N-2) \cdots 2 \cdot 1! = 1$, and by definition $0! = 1$ as well.
18. Complete the "ln Ω" column. ln X means the "natural logarithm of X:" when e is raised to the power of X, the result is $\ln(e^x) = X$. Most values of Ω will be too large to compute exactly. Instead, use a spreadsheet or Stirling's approximation to calculate

ln Ω. The spreadsheet uses Excel's GAMMLN function to calculate $\ln[\Gamma(n+1)]$, where $\Gamma(n) = (n-1)!$.

Table. Locations of 100 dice in 5 zones.

Step	Dice in Each Zone					ln Ω
	1	2	3	4	5	
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

1. Make a line graph of the number of dice in each zone from the run. (Or just look at the graph that your spreadsheet makes.) How does the distribution of dice change over time?
2. Make a line graph of ln Ω from the run. (Or just look at the graph that your spreadsheet makes.) Overall, how did ln Ω change with time?
3. In either run, if ln Ω ever decreased, how large was the numerical value of the change?