PHYS 1110 Gravity Lesson

Example problems

Some constants we'll need are

$G = 6.672 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2}$
Mass of Earth $= 5.976 \times 10^{24}$ kg
Radius of Earth $= 6378$ km

1. Earth's surface gravitational field

$$g = GM/r^2 = (6.672 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})(5.976 \times 10^{24}\,\mathrm{kg})/(6.378 \times 10^6\,\mathrm{m})^2 = 9.80\,\mathrm{N/kg}$$

2. Earth's escape speed

$$\begin{aligned}
1/2\,mv^2 - \frac{GMm}{R} &= 1/2\,m0^2 - \frac{GMm}{\infty} \\
1/2\,mv^2 - \frac{GMm}{R} &= 0 \\
1/2\,mv^2 &= \frac{GMm}{R} \\
1/2\,v^2 &= \frac{GM}{R}
\end{aligned}$$

$$\begin{aligned}
v^2 &= \frac{2GM}{R} \\
&= \frac{2(6.672 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})(5.976 \times 10^{24}\,\mathrm{kg})}{6.378 \times 10^6\,\mathrm{m}} \\
&= 1.2503 \times 10^8 (\mathrm{m/s})^2 \\
v &= \sqrt{1.2503 \times 10^8 (\mathrm{m/s})^2} \\
&= 11.2 \times 10^3\,\mathrm{m/s}
\end{aligned}$$

That speed is mach 33, but only 3.73×10^{-5} times the speed of light. Very fast, but not at all relativistic.

3. Orbital speed

$$\begin{aligned}
\text{Centripetal force} &= \text{Gravitational force} \\
\frac{mv^2}{r} &= \frac{GMm}{r^2} \\
v^2 &= \frac{GM}{r} \\
v &= \sqrt{GM/r}
\end{aligned}$$

4. Orbital period

$$\begin{aligned}
v^2 &= \frac{GM}{r} \\
\left(\frac{2\pi r}{T}\right)^2 &= \frac{GM}{r} \\
\frac{4\pi^2 r^2}{T^2} &= \frac{GM}{r} \\
\frac{4\pi^2 r^3}{GM} &= T^2 \\
T^2 &= \frac{4\pi^2}{GM} r^3
\end{aligned}$$

5. Uranus's moon Miranda

The planet Uranus has a radius of 25,362 km and a surface gravity of 8.87 N/kg at its poles. Its moon Miranda is in a circular orbit at a distance of 129,560 km from Uranuss center. Miranda has a mass of 6.6×10^{19} kg and a radius of 235 km.

a. Calculate the mass of Uranus from the given data.

$$g = \frac{GM}{r^2}$$

$$\begin{aligned}
M &= gr^2/G \\
&= (8.87\,\mathrm{m/s^2})(25.362 \times 10^6\,\mathrm{m})^2/(6.672 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}) \\
&= 8.55 \times 10^{25}\,\mathrm{kg}
\end{aligned}$$

b. Calculate the magnitude of Mirandas orbital acceleration due to its orbital motion about Uranus.

$$\begin{aligned}
g_2 &= \frac{GM}{r_2^2} \\
&= \frac{(Ggr^2/G)}{r_2^2} \\
&= g(r/r_2)^2 \\
&= (8.87\,\mathrm{m/s^2})(25,362/129,560)^2 \\
&= 0.340\,\mathrm{m/s^2}
\end{aligned}$$

c. Calculate the acceleration due to Mirandas gravity at the surface of Miranda.

$$g_3 = GM/r^2 = (6.672 \times 10^{-11}\,\mathrm{M \cdot m^2/kg^2})(6.6 \times 10^{19}\,\mathrm{kg})/(235,000\,\mathrm{m}) = 0.0797\,\mathrm{m/s^2}$$

Notice here that at the surface of Miranda the gravitational pull from Uranus is substantially stronger than the gravitational pull from Miranda.

d. Do the answers to parts b and c mean that an object released 1 m above Mirandas surface on the side toward Uranus will fall *up* relative to Miranda? Explain what is happening.

The rock *will* accelerate toward Uranus at the rate of $0.340 - 0.080 = 0.260\,\mathrm{m/s^2}$. But Miranda is accelerating toward Uranus at the rate of $0.340\,\mathrm{m/s^2}$. However you look at it, the distance between the rock and Miranda will close at the rate of $0.080\,\mathrm{m/s^2}$.