PHYS 1120 Discussion 1

Brief Solutions

1. Hydraulic system

a. What is the force on piston 2?

Pressure is force divided by area, so force is pressure times area, $F_2 = p \cdot A_2$. We can find the pressure from piston 1, for which both force and area are given. We are also given the area of piston 2. This yields $F_2 = (F_1/A_1)A_2 = F_1(A_2/A_1) = (50\text{ N})(1000/100) = 500\text{ N}$.

b. How far does piston 2 rise?

We know from conservation of energy that the work is the same on both pistons, so $F_1\Delta x_1 = F_2\Delta x_2$. We could also come at this from the incompressibility of the hydraulic fluid, which requires that the volume of fluid leaving cylinder 1 equals the volume of fluid entering cylinder 2.

$$\begin{aligned} -\Delta V_1 &= \Delta V_2 \\ -A_1\Delta x_1 &= A_2\Delta x_2 \\ \Delta x_2 &= -\Delta x_1(A_1/A_2) \\ \Delta x_2 &= -(-10\text{ cm})(100/1000) = 1.0\text{ cm} \end{aligned}$$

2. Garage hydraulic lift

a. What is the cylinder pressure?

Pressure is force divided by area. Here the force is the weight of the truck, $F_1 = mg = (2000\text{ kg})(9.8\text{ N/kg}) = 19,600\text{ N}$. To find the pressure, divide by the area of the piston, $p = 19,600\text{ N}/(500\text{ cm}^2) = 39.2\text{ N/cm}^2$. That sounds reasonable, but it isn't in SI units. We need square meters in the denominator, not square centimeters. Since $100\text{ cm} = 1\text{ m}$, $10,000\text{ cm}^2 = 1\text{ m}^2$, so the pressure in SI units is 392,000 Pa.

b. Gauge pressure or absolute pressure?

This is gauge pressure.

c. What is the absolute pressure?

Absolute pressure is gauge pressure plus the ambient pressure, or 392,000 Pa + 101,325 Pa = 493,325 Pa. Of course, we don't have that many significant figures, so it's more like 493,000 Pa = 4.9 bar.

d. Force on the driving piston

This will be the gauge pressure times the piston area, or the output force times the area ratio, $F_2 = F_1(A_2/A_1) = (19,600\text{ N})(10/500) = 392\text{ N}$.

e. Distance traveled by the driving piston

Since the area ratio is 50, the driving ratio must travel 50 times the distance as the output piston, or $(1.8\text{ m}) \cdot 50 = 90\text{ m}$.

3. Challenger deep

a. Gauge pressure at depth

Pressure $p = \rho g h = (1025 \text{ kg/m}^3)(9.8 \text{ N/kg})(10920 \text{ m}) = 109691400$ Pa. That is over 1000 bar.

b. Gauge vs. absolute pressure

The difference between gauge and absolute pressure here is less than a part per thousand, so it is not significant.

4. Earth's atmosphere

a. Weight of a 1 m^3 column of air

The pressure is the weight divided by the area, so the weight is the pressure times the area. The pressure is 101325 Pa =101325 N/m^2 and the area is 1 m^2), so the weight is 101325 newtons.

b. Thickness of the atmosphere

The thicker the atmosphere at a constant density, the heavier it will be. The atmosphere's weight is mg. The mass m we can find from density and volume, $m = \rho V$. The volume of a column is its cross-sectional area times its height, $V = Ah$. Here, we want to find the height h. We start with weight $w = mg = \rho V g = \rho A h g$. We can solve this to find h, as all other quantities are known here.

$$w = \rho A h g$$
$$h = \frac{w}{\rho A g}$$
$$h = \frac{101325 \text{ N}}{(1.204 \text{ kg/m}^3)(1 \text{ m}^2)(9.8 \text{ N/kg})}$$
$$h = 8,587 \text{ m}$$

We could have simplified the substitution a little by replacing the weight $w = pA$, which would have given us

$$pA = \rho A h g$$
$$h = \frac{p}{\rho g} = \frac{101325 \text{ N/m}^2}{(1.204 \text{ kg/m}^3)(9.8 \text{ N/kg})}$$
$$h = 8,587 \text{ m}.$$