PHYS 1120 Discussion 2: More with Fluids

Summary

Buoyancy

The upward force on a body inside an incompressible liquid is equal in magnitude to the weight of liquid displaced, ρVg. It the body is only partly submerged, V is the submerged volume.

Continuity in steady-state flow

If no fluid enters or leaves the channel, and if the density is steady with time, the mass flow rate of fluid is the same through all cross-sections.

$$\Delta m_1/\Delta t = \Delta m_2/\Delta t$$

If the density of the fluid is the same everywhere, as happened when the fluid is incompressible, then the volume flow rate is the same through all cross-sections.

$$\Delta V_1/\Delta t = \Delta V_2/\Delta t$$

Bernoulli's equation

For a non-viscous incompressible fluid with the continuity conditions above, conservation of mechanical energy requires

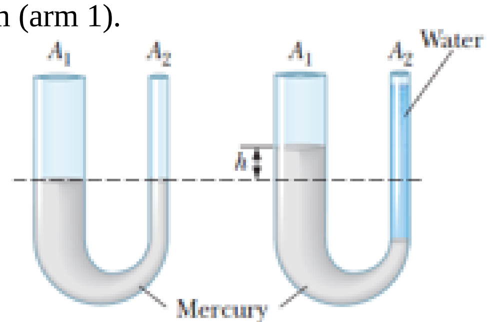
$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

where p = pressure h, v = fluid speed, ρ = density of the fluid, and y = height.

Problems

There is not space on the sheet to work the problems. Use scratch paper.

1. A U-tube containing liquid mercury, density ρ_M, has arms of different cross-sectional areas A_1 and A_2. A volume V_2 of water is added to the arm with area A_2. The water floats on top of the mercury, pushing the mercury column downward beneath it (arm 2) and pushing the mercury column upward in the other arm (arm 1).



   a. What is the height of the water column in arm 2?

   b. What is the pressure at the bottom of the water column in arm 2?

   c. Is this gauge pressure or absolute pressure?

   d. When the mercury column in arm 2 is pushed down a distance Δy_2, what volume of mercury ΔV_2 is pushed out of the arm?

   e. When a volume ΔV_2 of mercury is pushed out of arm 2, what volume ΔV_1 of mercury is pushed into arm 1?

f. When the column of mercury in arm 2 is pushed down a distance Δy_2, what distance Δy_1 does the surface of the mercury column in arm 1 rise?

g. How high above the bottom of the water column in arm 2 is the top of the mercury column in arm 1?

h. The pressure at the bottom of the water column in arm 2 must equal the pressure at the same height in column 1. Through what height $h = \Delta y_1$ does the mercury column in arm 1 rise when the water is added atop the mercury in column 2?

2. A layer of carbon tetrachloride, density ρ_1 = 1,594 kg/m^3, is poured into in a cylinder. A layer of water, density ρ_2 = 1,000 kg/m^3, is carefully poured on top of it. A sample with volume V = 2.00 cm^3 of the engineering plastic Delrin, density ρ_3 = 1,410 kg/m^3, is placed in the container, where it floats at the interface between the two liquids.

   a. At equilibrium, a volume fV is below the surface of the carbon tetrachloride layer. What volume is above the surface of the carbon tetrachloride layer?

   b. What is the buoyancy force on the Delrin from the carbon tetrachloride?

   c. What is the buoyancy force on the Delrin from the water?

   d. What is the total buoyancy force on the Delrin?

   e. What fraction f of the Delrin's volume is below the surface of the carbon tetrachloride layer?

3. The pulmonary artery carries all the blood from the heart into the lungs. The volume flow rate of a resting human is about 5 liters per minute, increasing to about 25 liters per minute during vigorous exercise. The diameter of the pulmonary artery is about 32 millimeters. Blood has a density of 1060 kg/m^3; 1 cubic meter = 1000 liters.

   a. What are the volume flow rates in cubic meters per second?

   b. What is the cross-sectional area of the pulmonary artery in square meters?

   c. What are the two flow speeds in meters per second?

   Suppose that a patient has a blockage in part of the pulmonary artery which reduces its cross section by half. Suppose further that the patient's blood pressure is 130 millimeters of mercury.

   d. What is the patient's blood pressure in pascals?

   e. Is the 130 millimeters of mercury gauge pressure or absolute pressure?

   f. What are the two flow speeds past the blockage?

   g. What is the pressure of blood flowing past the blockage when the patient is resting?

   h. What is the pressure of blood flowing past the blockage when the patient exercises vigorously?