PHYS 1120 Discussion 2

Brief Solutions

1. U-Tube with mercury and water

a. Height of the water column

Volume is area $\cdot$ height $= V = Al$, so $l = V/A = V/A_2$.

b. Pressure at the base of the water column

$p = \rho g l = \rho_w g V/A_2$.

c. Type of pressure measurement

This is gauge pressure.

d. Expelled volume from arm 2

$\Delta V_2 = \Delta y_2 A_2$.

e. Volume of mercury introduced to arm 1

There is no other place for the mercury to go, so $\Delta V_1 = \Delta V_2$.

f. Height increase in arm 1

Volumes are the same, but heights depend on volume and cross sectional area. $\Delta V_1 = \Delta y_1 A_1$ and $\Delta V_2 = \Delta y_2 A_2$, so

$$\begin{aligned} \Delta V_1 &= \Delta V_1 \\ \Delta y_1 A_1 &= \Delta y_2 A_2 \\ \Delta y_1 &= \Delta y_2 A_2 / A_1 \end{aligned}$$

g. Mercury column height

The top of the mercury column in arm 1 was pushed up a distance Δy_1, while the top of the mercury column in arm 2 was pushed down a distance Δy_2. So the height difference between the mercury columns in arm 1 and arm 2 is $\Delta y_1 + \Delta y_2$.

h. Height rise in column 1

Here we're looking for $h = \Delta y_1$. The clue we are given is to match pressures at a particular depth on both sides. The column of mercury above that depth in arm 1 has a height of $\Delta y_1 + \Delta y_2$; the column of water above that depth in arm 2 has a height of V/A_2. Equalizing pressures gives

$$\begin{aligned} \rho_m g(\Delta y_1 + \Delta y_2) &= \rho_w g V/A_2 \\ \rho_m g(\Delta y_1 + \Delta y_1 A_1/A_2) &= \rho_w g V/A_2 \\ \rho_m \Delta y_1 (1 + A_1/A_2) &= \rho_w V/A_2 \\ \Delta y_1 \frac{A_1 + A_2}{A_2} &= \frac{\rho_w}{\rho_m} \frac{V}{A_2} \\ \Delta y_1 &= \frac{\rho_w}{\rho_m} \frac{V}{A_2} \frac{A_2}{A_1 + A_2} \\ \Delta y_1 &= \frac{\rho_w}{\rho_m} \frac{V}{A_1 + A_2} \end{aligned}$$

2. Floating at the Interface

a. Fraction above bottom liquid

The fraction below the liquid id f, so the fraction above is $1 - f$.

b. Buoyancy from carbon tetrachloride

Let's call the volume of the Delrin sample V. Then the volume of carbon tetrachloride it displaces is fV. The weight of that volume is $\rho_1 g f V$.

c. Buoyancy from water

The volume of water displaced is $(1 - f)V$. The weight of that volume of water is $\rho_2 g(1 - f)V$.

d. Total buoyancy force

$\rho_1 g f V + \rho_2 g(1 - f)V = gV[\rho_1 f + \rho_2(1 - f)]$

e. Fraction f

The total buoyancy force must equal the Delrin sample's weight, because the system is static.

$$\begin{aligned} \rho_3 g V &= gV[\rho_1 f + \rho_2(1 - f)] \\ \rho_3 &= \rho_1 f + \rho_2 - \rho_2 f \\ \rho_3 - \rho_2 &= (\rho_1 - \rho_2) f \\ f &= \frac{\rho_3 - \rho_2}{\rho_1 - \rho_2} \end{aligned}$$

This formula makes intuitive sense; $f = 0$ if Delrin has the density of water, and $f = 1$ if Delrin has the density of carbon tetrachloride.

3. Pulmonary artery

a. Volume flow rate conversion

$$5 \frac{\text{L}}{\text{min}} \cdot \frac{1 \text{ m}^3}{1000 \text{ L}} \cdot \frac{60 \text{ s}}{\text{m}} = 8.33 \times 10^{-5} \text{ m}^3/\text{s}$$

Under vigorous exercise, the volume flow rate is five times that, or $4.17 \text{ m}^3/\text{s}$.

b. Area calculation

A 32 millimeter diameter is a radius of 16 millimeters = 0.016 meter. The area is then $\pi r^2 = 8.04 \times 10^{-4} \text{ m}^2$.

c. Flow speeds

Volume flow rate $\Delta V/\Delta t$ and flow speed v are related by area A: $\Delta V/\Delta t = vA$. This gives flow speeds of 0.1036 m/s and 0.5182 m/s.

d. Pressure conversion

Atmospheric pressure is 760 torr (millimeters of mercury) or 101,325 pascals. THis allows us to convert 130 torr = 17,332 pascals.

e. Gauge or absolute pressure?

This is gauge pressure; absolute pressure must be more than atmospheric.

f. Flow speed in restricted channel

In the restricted channel, the speed will be twice the speed in the normal channel. That means 0.2072 and 1.036 m/s.

g. Pressure in restricted channel

For this, we use the Bernoulli equation. There is no elevation difference in the different sections, so $y_2 = y_1$. We also know that the speed in the restricted channel is twice the speed in the normal channel, $v_2 = 2v_1$.

$$p_1 + 1/2\ \rho v_1^2 + \rho g y_1 = p_2 + 1/2\ \rho v_2^2 + \rho g y_2$$
$$p_1 + 1/2\ \rho v_1^2 + \rho g y_1 = p_2 + 1/2\ \rho (2v_1)^2 + \rho g y_1$$
$$p_2 = p_1 + 1/2\ \rho v_1^2 - 2\rho(2v_1)^2$$
$$p_2 = p_1 - 3/2\ \rho v_1^2$$
$$p_1 - p_2 = 3/2\ \rho v_1^2$$

For the patient at rest, this gives a pressure difference $p_1 - p_2$ of 17.1 pascals. This gives a gauge pressure in the restricted channel of 17,315 pascals = 130 torr.

h. Restricted channel pressure during exercise

This requires the same formula used in the previous section, just with a five-fold greater flow speed. This gives a pressure difference of 427 pascals, and a gauge pressure of 16,910 pascals = 127 torr.

Either way, the pressure drop due to the blockage is not substantial. Accounting for viscosity and turbulence is more likely to reveal a problem with arterial blockage. But we see that understanding the physics of the human body is not trivial.