KEY

PHYS 1120 Discussion 8.
Phasors

In an alternating current (ac) circuit, the volage from the power supply is $V = V_S \sin(\omega t)$. This can be visualized as the y-component of a vector of constant magnitude V_S rotating at constant angular speed ω, so that its angular position at time t is ωt. Such a vector is known for short as a **phasor**.

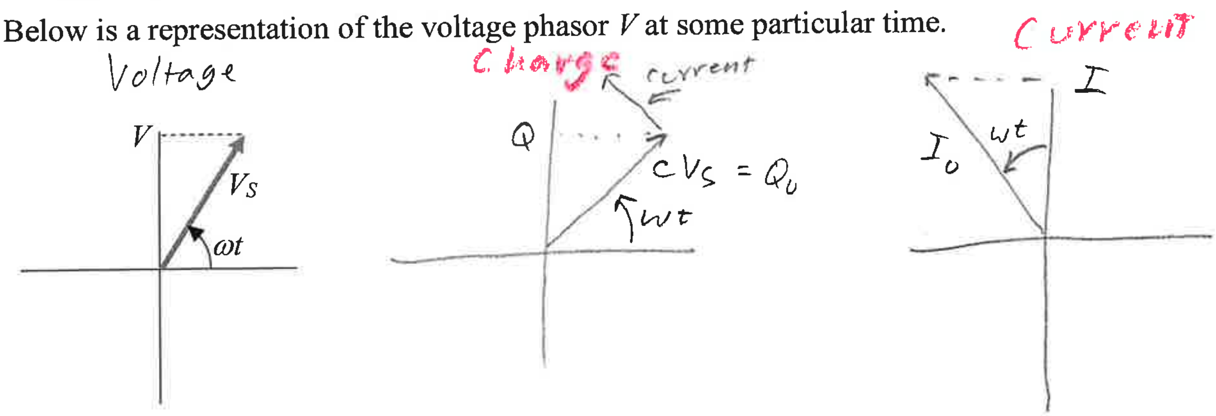
Resistors

In a resistor, Ohm's law tells us that the current is directly proportional to the voltage: $I = V/R$. This, for a resistor, the current phasor I always points in the same direction as the voltage phasor V_R: the current and voltage are *in phase*.

Capacitors

In a capacitor, charge, not current, is proportional to voltage: $Q = CV$. This means that the charge phasor Q is in phase with the voltage phasor V. The current, on the other hand, is the rate of change of the charge: $I = \Delta Q/\Delta t$. We can think of $\Delta Q/\Delta t$ as the velocity of the phase vector Q. What we know of uniform circular motion tells us that the speed of something rotating at angular speed ω in a circular path of radius r is ωr. Here, the current Q is rotating at angular speed ω with radius $Q_0 = CV_S$. Its speed must be $\omega Q_0 = \omega CV_S$.

Below is a representation of the voltage phasor V at some particular time.



1. Draw a representation of the charge phasor Q at that same time.
2. What is the magnitude Q_0 of the charge phasor Q? $Q_0 = CV_S$
3. Draw a representation of the current phasor I at that time. Remember that the current phasor is the velocity of the charge phasor.

Notice that the current phasor *leads* the voltage phasor by ¼ cycle.

4. What is the magnitude I_0 of the current phasor? $I_0 = \omega CV_S$
5. What is the formula for the y-component of the current phasor at time t? (The current is just the y-component of the current phasor.) $I = \omega CV_S \cos(\omega t)$

We'd like to extend Ohm's law to apply to the voltage and current in a capacitor. As Ohm's law tells us $V = IR$, we'd like to define a quantity of the capacitor, its reactance X_C, that relates its current and voltage as $V = IX_C$.

6. What is the reactance of a capacitor with capacitance C in a circuit whose voltage varies at angular frequency ω? $V_s = X_C I_0 = X_C \omega C V_s$, so $\boxed{X_C = \frac{1}{\omega C}}$

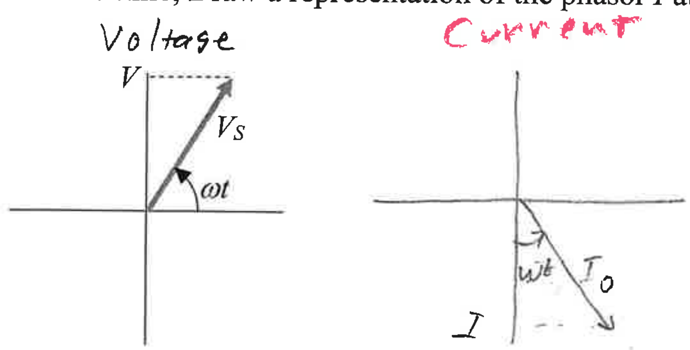
Inductors

How about an inductor? In an inductor, the voltage $V_L = L\,\Delta I/\Delta t$, so V_L/L is the velocity of the current phasor I. We want to find the magnitude of I. This will require some backward analysis.

In uniform circular motion, speed v is related to radius R and angular speed ω as $v = \omega R$. If we know v and ω, we can find $R = v/\omega$. Here, we have a speed of V_L/L and an angular speed of ω. $I = (V_L/L)/\omega = V_L/(\omega L)$

7. What is the "radius" I_0? This value is the maximum current through the inductor, or the magnitude of the current phasor. Simplify the expression. $I_0 = V/(\omega L)$

8. What is the direction of phasor I? Below is a representation of the voltage phasor V at some time; Draw a representation of the phasor I at that time.



9. What is the formula for the y-component of the current phasor at time t? (The current is just the y-component of the current phasor.) $I = -I_0 \cos(\omega t) = \frac{-V}{\omega L}\cos(\omega t)$

We'd like to extend Ohm's law to apply to inductors, defining inductive reactance X_L much as capacitive reactance above: $V = IX_L$.

10. What is the reactance of a inductor with inductance L in a circuit whose voltage varies at angular frequency ω? $X_L = V/I = \boxed{\omega L}$

Notice that the current phasor in an inductor *lags* the voltage phasor by ¼ cycle.

Reactance and Resistance

When a resistor is in series with a capacitor or inductor in an ac circuit, the current is always the same through all components, though their voltages are in different phases. I don't want to slog through that trigonometry now, but what we know gives us a way to find the reactance of a capacitor or inductor if we can measure the rms voltages of both it and the resistor in series. Because the current is the same, $I = V_R/R = V_X/X$, so $X = R\,V_X/V_R$.

Reactance and Resonance

An ac circuit will **resonate** when the capacitive reactance equals the inductive reactance.

11. At what angular frequency ω is the inductive reactance X_L of an inductor with inductance L equal to the capacitive reactance X_C of a capacitor with capacitance C?

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$