PHYS 1120 Discussion 11. Interference and Diffraction of light

Brief Solutions

1. Double slit diffraction

A. Angles of the first four bright fringes

In double slit diffraction, bright fringes are at angles θ given by $\sin\theta = m\lambda/d$, where m is an integer and d is the slit spacing. So here we look for the angles θ corresponding to $m = 1$–4. Specifically, $\lambda = 633 \times 10^{-9}$ m and $d = 0.5 \times 10^{-3}$ m.

m	$\sin\theta = m\lambda/d$	θ (degrees)
1	0.001266	0.0725
2	0.002632	0.145
3	0.003798	0.218
4	0.005064	0.290

B. Slit spacing for 5-degree first fringes

We have the same equation $\sin\theta = m\lambda/d$, but now we know θ and want to find a corresponding d. $d = m\lambda/\sin\theta = (633\text{ nm})/0.08176 = 7268\text{ nm} = 7.27\ \mu\text{m}$. This is much closer than the original slit spacing. The closer together the slits are, the more spread out the diffraction pattern will be.

2. Single slit diffraction

A. Dark lines in the pattern

For diffraction of waves with wavelength λ through a single slit of width d, the dark bands are at angles θ given by $\sin\theta = m\lambda/d$.

B. Angles of the nodes ("dark bands")

m	$\sin\theta = m\lambda/d$	θ (degrees)
1	0.2	11.5
2	0.4	23.6
3	0.6	36.9
4	0.8	53.1
5	1.0	90

There cannot be any more nodes than this, because the sine of an angle cannot exceed 1.

3. Oil film on water

A. Wavelength in another medium

Light travels more slowly in a medium of a different index of refraction, but its frequency of oscillation does not change. So $v = \lambda f = c/n$ in the medium and $c = \lambda_0 f$ in air. Thus $\lambda = c/(nf) = \lambda_0/n$.

B. Phase advance on traveling

Traveling a distance d is traveling d/λ wavelengths. The phase advance of traveling one wavelength is 2π, so the phase advance of traveling d/λ wavelengths is $2\pi d/\lambda$.

C. Phase advance of the first reflected beaml

This is the light reflecting off the top surface of the oil. The index of refraction of oil is higher than the index of refraction of air, so the light changes phase by π radians upon reflection.

D. Wavelengths traveled through oil

We only know the wavelength as λ: we don't know its actual distance. But when the light travels a distance d, the number of wavelength distances is $d/\lambda = dn/\lambda_0$.

E. Phase advance in oil

If the light advances dn/λ_0 wavelengths, its phase advance is that number times 2π, the phase advance of one full cycle. So it's $2\pi dn/\lambda_0$.

F. Phase change reflecting off water

The index of refraction of water is less than the index of refraction of oil, so the light does not change phase upon reflection.

G. Phase advance traveling back up through oil

The same as passing through the oil on the way down, $2\pi dn/\lambda_0$.

H. Phase change of second reflected beam

This is the sum of its advances traveling down through the oil, reflecting off the water, and traveling back up through the oil: $2\pi dn/\lambda_0 + 0 + 2\pi dn/\lambda_0 = 4\pi dn/\lambda_0$.

I. Formula for wavelengths with constructive interference

The phase difference $\varphi_2 - \varphi_1 = 4\pi dn/\lambda_0 - \pi$. For this to give constructive interference, the phase difference must be an integer multiple of 2π.

$$\begin{aligned} 4\pi dn/\lambda_0 - \pi &= 2\pi m \\ 2nd/\lambda_0 - 1/2 &= m \\ 2nd &= \lambda_0(m + 1/2) \\ \lambda_0 &= \frac{2nd}{m + 1/2} \end{aligned}$$

This formula is a little surprising, showing that the wavelength of reinforced light can be much longer than the thickness of the film. This is due to two reasons: first, because the wavelength is shortened in the film compared to in air, and more importantly, because the path length added by traveling through the film (twice!) only needs to be half of that wavelength.

J. Visible wavelengths with constructive interference

This formula We'll list the first bunch of wavelengths, and then pick the ones in the visible range.

m	$\lambda_0 = 2nd/(m + 1/2)$
0	4560 nm
1	1520 nm
2	912 nm
3	651 nm
3	506 nm
3	414 nm
3	350 nm

We can stop here, because any further wavelengths will be in the ultraviolet range. The visible wavelengths in this series are at 651 (red), 506 (cyan), and 414 nm (violet).