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PHYS 1120 Discussion 12.
The Twin Paradox

Review

Three key predictions of Special Relativity show the interplay between time and space.

The fundamental postulate of Special Relativity is that the laws of physics are the same in all inertial reference frames. These laws of physics include that the vacuum speed of light is c. This requirement has anti-intuitive consequences.

Notation that will be used frequently in this exercise include $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. These parameters are dimensionless: β ranges between zero and 1 (or between –1 and 1), and γ ranges between 1 and infinity.

Time dilation

When a body travels with constant velocity $\vec{v}$ relative to an inertial "laboratory" frame of reference, any process it undergoes with duration t' in its rest frame has duration $t = \gamma t'$ in the laboratory frame.

Length contraction

A body traveling with speed v will be shortened along its direction of travel. If its rest length along its direction of travel is L_0, in the laboratory frame its length is $L = L_0/\gamma$.

Relativistic Doppler effect

Electromagnetic radiation with frequency f_S in the rest frame of its emitter is received with frequency

$$f_D = f_S\sqrt{\frac{1+\beta}{1-\beta}}$$

where the emitter travels toward the detector with speed v.

The Twin Paradox

Perry and Quinn are twins. Quinn, an astronaut, embarks on a voyage to the star δ-Pavonis, 19.9 light-years distant. For simplicity, let's round it to 20 ly distant. Quinn travels to δ-Pavonis at a speed of $v = \sqrt{3}/2\ c \approx 0.866\ c$, and once there, immediately turns around and travels back to Earth at the same speed.

1. According to Perry, how much time does it take for Quinn to reach δ-Pavonis?
2. According to Perry, how long does it take Quinn to travel from δ-Pavonis back to Earth?
3. According to Perry, how long after leaving does Quinn return to Earth?

4. Traveling out and back at $\sqrt{3}/2\ c$, what is Quinn's time dilation (γ)? In other words, for each year that passes for Quinn, how many years pass for Perry?
5. As Quinn travels to δ-Pavonis, it appears that δ-Pavonis approaches at a speed of $\sqrt{3}/2\ c$. This means that the distance from Earth to δ-Pavonis appears, to Quinn, shorter by a factor of γ. In Quinn's frame of reference, what is the distance from Earth to δ-Pavonis?
6. In Quinn's frame of reference, how much time will it take to travel to δ-Pavonis?
7. In Quinn's frame of reference, how much time will it take to travel from δ-Pavonis back to Earth?
8. In Quinn's frame of reference, how much time does the round trip between Earth and δ-Pavonis take?
9. How many years have passed for Perry and Quinn when they reunite?

This is part of the paradox. When they reunite, Quinn is much younger than Perry: the time they experience between the same events, departure and return, is different!

You already determined that while Quinn is traveling, according to Perry, each year passing for Quinn takes __________________ for Perry.

10. What about Quinn's perspective? While Quinn is traveling at a speed of $\sqrt{3}/2\ c$, as far as Quinn is concerned, *Perry* is traveling at $\sqrt{3}/2\ c$, first away, then back. From Quinn's perspective, *Perry's* time is dilated. In Quinn's frame of reference, for each year passing for Perry, how many years pass for Quinn?

This is the other part of the paradox. If each twin is slowed down relative to the other, why does Quinn experience less time passing than Perry?

Once each year, Perry radios a birthday message to Quinn. Once each year, Quinn radios a birthday message to Perry. You can think of the messages sent between the twins as signals with a frequency of one cycle per year.

11. Quinn sends birthday messages at a frequency of one message per year. Perry receives these messages Doppler shifted. With what frequency does Perry receive Quinn's annual messages?

12 Perry sends birthday messages at a frequency of one message per year. Quinn receives these messages Doppler shifted. With what frequency does Quinn receive Perry's annual messages?

13. You should have found the same frequency of messages received by both twins. What is the period—the time interval between messages?
14. As Quinn travels toward δ-Pavonis, how many birthday messages arrive from Perry?
15. How many birthday messages does Quinn send to Perry on the way to δ-Pavonis?

Upon reaching δ-Pavonis, Quinn radios an "I'm turning around" message to Perry.

16. According to Perry, how much time after Quinn's departure is the "I'm turning around" message sent?
17. How much time after Quinn's departure does the "I'm turning around" message reach Perry?
18. While Quinn travels back to Earth from δ-Pavonis, the birthday messages to both Perry and Quinn are Doppler shifted the other way. With what frequency do Perry and Quinn receive each other's annual messages?
19. What is the period—the time between receiving successive annual messages—during Quinn's return journey?
20. How many of Perry's annual messages does Quinn receive on the return journey?
21. How many of Quinn's annual messages sent from the return journey does Perry receive?
22. Over what length of time does Perry receive Quinn's messages sent on the return journey?

We see that one fundamental asymmetry of this scenario is that Perry receives the same number of frequent messages from Quinn as infrequent messages, and the frequent messages all arrive in a much shorter time period. Quinn receives infrequent and frequent messages from Perry for the same amount of time, so many more of the messages are frequent.

Another fundamental asymmetry is that Perry occupies just one inertial reference frame throughout the entire scenario, while Quinn occupies two: one traveling away at velocity $\vec{v}$, and one returning at velocity $-\vec{v}$. If you make the situation more symmetrical—say, by having Perry follow Quinn to δ-Pavonis—they age the same.