# PHYS 1120 Discussion 12. Twin Paradox

Brief Solutions

**1. Quinn's outbound time, according to (Earthbound) Perry**

$\Delta t = \Delta x/v = (20 \text{ ly})/(\sqrt{3}/2 \text{ ly/y}) = 23.09$ y.

**2. Quinn's return time, according to Perry**

Same as the outbound time, 23.09 y.

**3. Quinn's round-trip time, according to Perry**

46.19 y

**4. Time dilation**

$\gamma = 1/\sqrt{1 - 3/4} = 2$. So a year for Quinn is two years to Perry.

**5. Earth–$\delta$-Pavonis separation, according to Quinn**

Quinn sees the distance shortened by a factor of $\gamma = 2$, so it becomes 10 light-years.

**6. Quinn's perceived outbound travel time**

10 light-years at $\sqrt{3}/2$ light years per year takes 11.55 years.

**7. Quinn's perceived return travel time**

Same as the outbound time, 11.55 years.

**8. Quinn's perceived round-trip time**

23.09 years, exactly half what Perry measures.

**9. Time passed**

For Perry, 46.19 years have passed. For Quinn, exactly half that time, 23.09 years.

**10. Perry's time dilation from Quinn's perspective**

Same as Quinn's time dilation from Perry's perspective: $\gamma = 2$, or one year passing for Perry is two years for Quinn.

**11. Doppler shifted annual messages to Quinn**

The greetings are sent with a frequency of $f_S =$ one message per year.

$$f_D = f_S\sqrt{\frac{1-\beta}{1+\beta}} = f_S\sqrt{\frac{1-\sqrt{3}/2}{1+\sqrt{3}/2}} = 0.2679 f_S$$

The greetings are received at a frequency of $f_D = 0.2679$ messages per year.

**12. Doppler shifted annual messages to Perry**

The formula and numbers are the same as the messages from Perry to Quinn. Perry receives Quinn's greetings at a frequency of $f_D = 0.2679$ messages per year.

**13. Period of messages received**

The period is the reciprocal of frequency, or one message every 3.732 years: 3.732 years between messages.

**14. Messages received by outbound Quinn**

(0.2679 messages/y)(11.55 y) = 3.094 messages). Yes, fractional messages don't make sense, but the point here is that Quinn, while traveling to $\delta$-Pavonis, receives signals from Perry corresponding to 3.094 years of Perry's life.

**15. Messages sent by outbound Quinn**

Quinn sends a message each year fro 11.55 years, so that's 11.55 messages. Again, fractional messages don't make sense, but the point is that the messages account for 11.55 years of Quinn's life.

**16. Timing of Quinn's "turn-around" message**

From Perry's perspective, Quinn takes 23.09 years to reach $\delta$-Pavonis, so that's when the message originates.

**17. Receipt of Quinn's "turn-around" message**

Once the message is sent, it travels 20 light-years at a speed of 1 light year per year to reach Perry. That adds up to 43.09 years.

**18. Doppler shift during approach**

With the directions reversed, the signs reverse in the Doppler formula.

$$f_D = f_S\sqrt{\frac{1+\beta}{1-\beta}} = f_S\sqrt{\frac{1+\sqrt{3}/2}{1-\sqrt{3}/2}} = 3.732 f_S$$

The greetings are received at a frequency of $f_D = 3.732$ messages per year.

**19. Period of messages received**

Again, the period is the reciprocal of frequency. Here, that is 0.2679 years between messages.

**20. Messages received by Quinn during return trip**

Receiving messages for 11.55 years at a rate of 3.732 messages per year gives 43.09 messages. That's 43.09 years of Perry's life. Note that this is exactly the amount of time that Perry waits to receive Quinn's "I'm turning around" message.

**21. Messages received by Perry from Quinn's return trip**

Perry receives 11.55 messages. Same disclaimer about fractional messages.

**22. Time Perry receives messages from Quinn's return trip**

This is the time between when Perry receives Quinn's "I'm turning around" message and when Quinn arrives, or 3.09 years. Note that this is exactly the number of messages (Perry years) that Quinn receives from Perry while traveling to $\delta$-Pavonis.