PHYS 1120 Discussion 13. Greatest Hits

Brief Solutions

1. Pressure

A. Pressures at the bottom

The water depths are the same, so the pressures at the bottom are the same.

B. Downward force by water

Force is pressure per area. The same pressure over the same area means it's the same force.

C. Weight of the vessels

The vessel on the right holds more water, so it is heavier.

D. Explain

In the vessel on the left, water pressure does not only push down on the base; it also pushes up on the slanted walls. So the weight is less than the pressure times the area of the base.

In the vessel on the right, water pressure pushes down on the slanted walls. Here, the weight is *more* than the pressure times the area of the base.

2. Dielectrics

A. Enhancing capacitance

If the gap between the plates of the capacitor is filed with a vacuum (air is almost the same), the electric field inside the gap is given by $E = \sigma/\epsilon_0 = Q/A\epsilon_0$. The potential difference between the plates is then the field multiplied by the gap thickness d, $V = Ed = Qd/A\epsilon_0$.

With the dielectric in the gap, the field is reduced by a factor of κ to $V = Qd/\kappa A\epsilon_0$. The capacitance is $C = Q/V = \kappa A\epsilon_0/d$. Following the same reasoning, the capacitance with only a vacuum dielectric is $A\epsilon_0/d$.

B. Capacitance by Gauss's law

We use Gauss's law to show that the electric field between two parallel plates with area charge density $\pm\sigma$ is $E = \sigma/\epsilon_0$. From there we find the voltage and capacitance.

3. Three DC circuits

In the circuit on the left, closing the switch does not change the current flow: it's 0.125 amperes either way. This means that no current flows through the bridge/switch. With the switch open, every resistor has a voltage of 2 volts. Closing the switch does not change that.

In the circuit in the center, closing the switch increases the total current substantially. With the switch open, the resistance of each branch of the circuit is 100 ohms, giving a current through each branch of 0.04 amperes and a total current of 0.08 amperes. Each 20 ohm resistor has 0.8 volts across it, and each 80 ohm resistor has 3.2 volts across it. When the switch closes, a lot of current will flow across the bridge from the 20 ohm resistor on top to the 20 ohm resistor on the bottom. Every resistor will have a voltage of 2.0 volts across it, for a current into each junction of $(2.0\ \text{V})/(20\Omega) + (2.0\ \text{V})(80\Omega) = 0.1\ \text{A} + 0.25\ \text{A} = 1.25\ \text{A}$.

In the circuit on the right, both 20-ohm resistors on top will have a voltage of 0.8 volts, and both 80-ohm resistors on the bottom will have a voltage of 3.2 volts. Closing the switch won't change the voltage, and it won't change the current, which is 1.25 volts.

4. Faraday's law

The magnetic field inside the solenoid (it's a "long" solenoid, so we can treat it as ideal with magnetic field uniform within the coil and zero outside) is given by the formula $B = \mu_0 n I$. We are given $n =$ 3500/m, but we aren't given I, only its rate of change $\Delta I/\Delta t = 28.5$ A/s. To calculate the emf from Faraday's law, we will need the rate of change of magnetic flux within the loop; the magnetic flux itself is $\Phi = BA = \mu_0 n I \pi r^2$. Again, we don't know I, but we only need to know the rate of change of magnetic flux, not the flux itself. Every quantity in the formula for Φ is constant except for I, so $\Delta\Phi/\Delta t = \pi r^2 \mu_0 n \ \Delta I/\Delta t = 1.58 \times 10^{-4}$ V.

5. Anti-reflective coating

We are told that the refractive index of the magnesium fluoride coating is 1.325 and of the glass is 1.530. The coating should prevent reflection of light with a wavelength of 550 nanometers.

A. Thickness of the coating

The coating has a higher refractive index than air and a lower refractive index than glass, so the reflected light will advance by half a cycle on both reflections. For cancellation, then, the condition is that the path length of the light inside the film is an odd number of half-wavelengths. We'll start with one half a wavelength, which means that the film must be a quarter wavelength thick ($1/4+1/4 = 1/2$). But the wavelength in question must be the wavelength of the light in magnesium fluoride, which is $\lambda/n =$ (550 nm)/1.325= 415.1 nm. One quarter of this is 103.8 nm.

B. Phase difference for 450-nm light

The phase difference will come from the path length of the light inside the coating. The coating is still 103.8 nanometers thick, and the path length of the light is still 207.5 nanometers. But the wavelength of this light in magnesium fluoride is 339.6 nm, so the path length is 0.61 wavelengths. It's not exactly the half-wavelength difference that is necessary for complete destructive interference, but the coating still will reduce reflection at that wavelength.

6. Replacement lens

A. Strength of replacement lens

We don't know where the actual object is, and we don't need to know. What we want to know now is how to correct the refraction from the cornea and aqueous humor so that the image falls at the retina, not behind it.

We are told that the distance from the cornea to the primary image is 5.00 cm, and from the cornea to the retina, where we want the image, is 2.60 cm. The converging lens will converge the rays from the cornea (and aqueous humor; I'm growing weary of typing that every time) more sharply, so that they converge to an image in a shorter distance. The object is not in front of the lens; it is behind it, so the object distance is negative. In the thin lens equation, then,

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{-5.00 \text{ cm}} + \frac{1}{2.60 \text{ cm}} = 0.1846/\text{cm}$$

This is, in fact, the answer that we seek: the lens strength is the reciprocal of its focal length. The only conversion that we need to do is to diopters, where one diopter is 1/m instead of the 1/cm that we have. Using 100 cm = 1 m, we obtain $1/f = 18.5/\text{m} = 18.5$ diopters.

B. Ray tracing

For the virtual object, light rays do not travel from it to the lens; instead, the rays would travel to the object if the lens were not in the way. But the lens converges the rays that would he headed for the virtual object. As always with lenses, three useful rays are the paraxial ray, which approaches the lens parallel to its axis, the principal ray, which passes through the center of the lens, and a focal ray, which travels to the lens from the direction of a focal point. The diagram below shows how these three rays work.