PHYS 1120 Exam 1

Brief Solutions

1. Jaws of Life

Pressure is force per area, $p = F/A$. Here F =60,000 N. Area isn't given, but we know the diameter. The area is $A = \pi r^2$ where the radius r is half the diameter, which we are told is 4.0 centimeters. So the area is 4π cm^2 and $p = 4.78 \times 10^7$ Pa = 478 bar = 4.78×10^3 N/cm^2.

2. Hydraulic pallet truck

We are told that the load piston moves 1 cm for every 12 cm that the pump piston moves.

A. Area of pump piston given area of the load piston

Calling the load piston 2 and the pump piston 1, we know from conservation of matter (and the fact that the hydraulic fluid is incompressible) that

$$\begin{aligned}\Delta V_1 &= \Delta V_2\\ A_1\Delta x_1 &= A_2\Delta x_2\\ A_1 &= A_2\Delta x_2/\Delta x_1\\ A_1 &= 2\text{ cm}^2\end{aligned}$$

B. Pump force

The load force is $F_2 = mg = (846\text{ kg})(9.8\text{ N/kg}) = 8291$ N. We know from Pascal's principle that

$$\begin{aligned}F_1/A_1 &= F_2/A_2\\ F_1 &= F_2A_1/A_2\\ F_1 &= (8291\text{ N})/12 = 691\text{ N.}\end{aligned}$$

C. Cylinder with higher pressure

Pascal's principle tells us that both cylinders have the same pressure as long as their heights aren't significantly different.

3. Ethane lake of Titan

We have everything we need to calculate pressure at depth,

$$p = \rho gh = (550\text{ kg/m}^3)(1.352\text{ N/kg})(5.5\text{ m}) = 4090\text{ Pa.}$$

4. Weather balloon

We are given mass of the balloon $m = 0.450$ kg, volume of the balloon $V = 3.40$ m^3, and the density of helium gas, $\rho_{\text{He}} = 0.171$ kg/m^3.

A. Mass of helium

$m = \rho_{\text{He}} V = (0.171 \text{ kg/m}^3)(3.40 \text{ m}^3) = 0.5814 \text{ kg}.$

B. Buoyancy force

For the buoyancy force, we need to use the density of air, not of helium.

$$F = \rho g V = (1.20 \text{ kg/m}^3)(9.8 \text{ N/kg})(3.40 \text{ m}^3) = 40 \text{ N}.$$

C. Payload mass

This is the mass that the balloon can lift. The weight of the payload will be the buoyancy force minus the weight of the balloon and helium; then we need to convert that weight to mass. The total mass of the balloon and helium is 1.0314 kg, which has a weight of 10.11 newtons. The payload weight then can be up to 40 N − 10.11 N = 29.88 N. this corresponds to a mass of 3.05 kilograms.

5. Pulmonary artery

We are given a volume flow rate $\Delta A/\Delta t = 4.17 \times 10^{-4}$ m^3/s, the cross sectional area of 8.094×10^4 m^2, and the density of blood of 1060 kg/m^3.

A. Speed

We use $\Delta V/\Delta t = vA$, so $v = A \cdot \Delta V/\Delta t = 0.519$ m/s.

B. Speed in a constriction

The volume flow rate is the same in the constriction as before it, so

$$v_1 A_1 = v_2 A_2$$
$$v_2 = v_1 A_1 / A_2$$
$$v_2 = 3v_1 = 1.56 \text{ m/s}$$

C. Pressure comparison

To understand this , we consult the Bernoulli equation

$$p_1 + 1/2 \rho v_1^2 + \rho g y_1 = p_2 + 1/2 \rho v_2^2 + \rho g y_2$$

We'll say 1 is before the constriction and 2 is at the constriction. We aren't told of any concerns with different heights before and at the constriction, so we can assume that $y_2 = y_1$. Then we have for the pressure at the constriction

$$p_2 = p_1 + 1/2\ \rho v_1^2 - 1/2\ \rho v_2^2$$

Because the speed at the constriction v_2 is three times the speed v_1 before the constriction, we know that $p_2 < p_1$.

D. Pressure p_2

Using the formula from the previous step. we get 15857 Pa.

6. Surface tension

These questions are conceptual/memorization problems with nothing to calculate.

A. Surface at higher or lower energy?

The surface is at hgigher energy than the bulk interior. Otherwise the molecules in bulk liquid would be more stable outside than inside the liquid, and it would not condense.

B. Units of surface tension

Force per length, which gives newtons per meter, This is equivalent to energy per surface area, which is joules per square meter. Mathematically, since joules are $\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$, this also is equivalent to $\mathrm{kg/s}^2$, though that does not provide much insight into the situation.

7. Viscosity

This is another set of questions that requires conceptual recall with no calculation.

A. Flow rate

Viscosity essentially created more drag on the stationary walls of the channel, resuoting in slower flow.

B. Turbulence

You can use a formula for this: higher Reynolds numbers mean a greater tendency to turbulence. The formula for Reynolds number gives lower numbers at higher viscosities, as viscosity is in the denominator. So higher viscodity means a lower tendency to turbulence.

8. Charges at the Moon

These questions require application of Coulomb's law and the associated electric potential formula

a. Charge giving repulsion of 1 newton at the Moons distance

Here we are told a force and asked for the charge. The force formula is Coulomb's law.

$$F = \frac{kq_1q_2}{r^2}$$

Here we are told both charges are the unknown quantity Q, and we are asled to find it.

$$F = kQ^2/r^2$$
$$Q^2 = Fr^2/k$$
$$Q = r\sqrt{F/k}$$

Applying this formula to $f = 3.844 \times 10^8$ m, $F = 1|rm\ N$, and $k = 8.988 \times 10^9\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$ gives 4055 coulombs.

B. Force at 1000 meters

Here we can take the charge we just found and the distance of 1000 m and plug them into Coulmb's law to find the force. We get 1.48×10^{11} newtons.

C. Electric field at Moon's distance

The field E is what would give a force of 1 newton on a charge of Q. From $F = QE$ we find $E = F/Q = 2.47 \times 10^{-4}$ N/C.

D. Electric potential at the Moon's distance

The formula for electric potential V created by a point charge Q is $V = kQ/r$. Using our numbers gives 94814 volts. Even though the Moon is far from Earth, Coulomb's constant is large and 4055 coulombs is a large charge when it comes to the electrical interaction force.

10. Electric polarization of an aluminum can

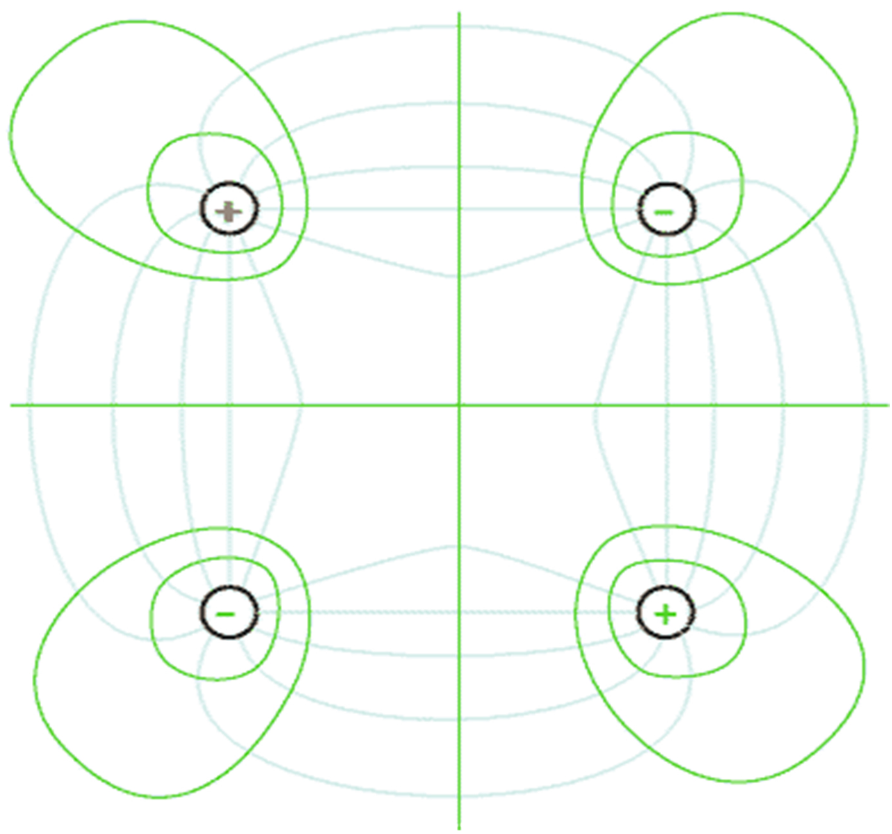
The key idea is that positive charges in the can are attracted to the negative charge of the balloon, while negative charges in the can are repelled from the balloon. Until the field inside the can becomes zero, positive charges thus will move toward the balloon and negative charges will move away. This results in a net force because the Coulombic force gets weaker with distance, so the repulsion from the more distant negative charges is less than the attraction to the closer positive charges. The closest choice to this explanation is c.

11. Incomplete field line map

We know the sign of the top left charge and no others. We see that field lines run from that positive charge to the adjacent top right and bottom left charges. Because field lines run from positive charges to negative charges, those two charges, those two charges must be negative. The same reasoning tells us that the lower right charge must be positive. The is all the information needed to answer parts A–8.

G. Isopotentials

Isopotentials are perpendicular to field lines, and, like field lines, evenly-spaced (in potential) lines are closer together where the field is stronger. In this diagram, that is near the charges. Ideally, an isopotential map shows closed loops around the charges that are more closely-spaced near the charges. In this diagram, there will be lines of zero potential bisecting the pairs of positive and negative charges, vertically and horixontally.



12. Conducting box in a uniform electric field

Because the box is made of a conductor, we know that charges inside it will rearrange until the electric field inside is zero.

A. Electric field inside the box

The electric field must be zero inside the box.

B. Face with the highest potential

Because the field is zero throughout the box, the potential is the same everywhere throughout the box.

13. Gauss's law for an infinite, straight wire of charge

The point of this problem is to apply Gauss's law $\Phi = EA = Q_{\text{in}}/\epsilon_0$.

A. Direction of the field

The field must point away from the positive charges in the wire. Because the wire is infinitely long, there cannot be any component in the vertical direction.

B. Charge enclosed Q_{in}

The Gaussian surface encloses a segment of the wire of length L. This segment carries a charge λL.

C. Flux through the surface

The flux through the surface is the electric field through the surface multiplied by the area of the surface. Gauss's law tells us that the flux is also the total charge contained within (enclosed by) the surface divided by the constant ϵ_0. That's what we use to answer this question.

D. Electric field strength

We want to find the strength of the field at the curved surface of the Gaussian cylinder. That is essentially the electric flux density there. There is zero flux through the end caps of the cylinder, because the field does not go through those surfaces. So the flux is EA, the field strength at the curved surface times the area of the curved surface, and also the multiple of the enclosed charge.

$$\begin{aligned} EA &= \frac{Q}{\epsilon_0} \\ E \cdot 2\pi r L &= \frac{\lambda L}{\epsilon_0} \\ E &= \frac{\lambda}{2\pi r \epsilon_0} \end{aligned}$$